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Television advertising's effect on the demand for different types of fresh beef: a Gibbs sampling approach

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Television advertising's effect on the demand for different types of fresh beef:

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This is to certify that the Master's thesis of

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has met the thesis requirements of Iowa State University

Signatures have been redacted for privacy

For the Graduate College

This thesis is dedicated to my wife Penny and my son Theron who have supported me throughout my research endeavors.

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INTRODUCTION

Major changes have occurred in the marketing of beef products in the United States since 1985 when the Beef Promotion and Research Act increased funding for promotion, advertising, and information activities. Per capita beef consumption had been declining in the United States, and the objective of the Act was to boost the demand for beef products by enhancing the consumer's image of beef (Jensen and Schroeter 1992). The purpose of this research is to investigate whether television advertising, in particular, has been successful in increasing the demand for different types of fresh beef products.

The data used in this study were obtained from a marketing research experiment done in Grand Junction, Colorado from 1985 to 1987. Jensen and Schroeter used these data in an econometric study of the effects of television advertising on aggregate fresh beef consumption. Their findings suggested that advertising actually had a small but statistically significant negative impact on household beef demand. The aggregation of all beef products into the single quantity variable used in their study may, however, have masked advertising's effects on consumption of specific types of beef. In addition, their analysis did not provide a unified treatment of the two key statistical aspects of the data set: its panel structure and a truncated distribution for the dependent variable.[1] The present study will investigate advertising's effects on demand for three types of fresh beef products; steaks, roasts, and ground beef;[2] and will do so using a Bayesian analysis of a random effects Tobit model suitable for a limited dependent variable/panel data application.

DATA AND EXPERIMENTAL DESIGN

The Grand Junction experiment was staged by Information Resources Incorporated (IRI) under contract to the Beef Industry Council of the National Live Stock and Meat Board. In this experiment, the beef purchases of approximately 2000 households were monitored for 92 weeks from October, 1985 to July, 1987. Each household was given an identification card to be shown when making purchases at area grocery stores. At checkout time of each shopping trip, the stores' UPC scanners read participants' beef purchases and used the information to update household purchase records throughout the experiment. The households also subscribed to cable television with advertisements that could be controlled on a household-by-household basis. Panel households were placed in one of three groups characterized by different levels of exposure to test advertisements. A "control" group received none of the test ads, a "base-ad" panel received moderate advertising, and a "heavy-ad" panel received extensive advertising. The first phase of the advertising experiment was a 16-week "pre-test" phase in which none of the three panels received any advertising. The second phase was a 48-week period in which the heavy-ad panel received a total of 4480 gross rating points (GRPs), and the base-ad panel received a total of 1220 GRPs of exposure to test ads from the "Beef Gives Strength" campaign.[3] The third phase was a 28-week period in which both the heavy-ad panel and the base-ad panel received a total of 1470 GRPs of exposure to test ads from the "Real Food for Real People" campaign. Again, the control group received no test advertising at any time throughout the experiment. Table 1 provides

Table 1. Time pattern of advertising intensity by panel

Phase	4-week Period	Advertising intensity in GRPs: base ad panel	heavy ad panel
Pretest	1	0	0
(16 weeks, 4 4-week	2	0	0
periods)	3	0	0
	4	0	0
Phase 1, ad test	5	360	720
(48 weeks, 12 4-week	6	180	360
periods)	7	0	680
	8	170	340
	9	170	340
	10	0	680
	11	0	340
	12	0	340
	13	340	680
	14	0	0
	15	0	0
	16	0	0
Phase 2, ad test	17	160	160
(28 weeks, 7 4-week	18	320	320
periods)	19	160	160
	20	550	550
	21	70	70
	22	210	210
	23	0	0

more detail about the distribution of advertising messages throughout the experiment's three periods.

The experiment's 92 weeks were divided into 23 four-week demand periods. For each of these periods, and for each panel household, the scanner data include total fresh beef expenditures (in cents) and total fresh beef purchased quantity (in pounds) for each of three product types: steaks, roast beef, and ground beef.[4] From these, category specific beef price indices can be inferred for each period by dividing panel-wide category-aggregate expenditures by panel-wide category-aggregate purchase quantities.

Seasonal adjustment factors are also included in the data set. These are based on the results of estimation of a national aggregate demand relationship using extraneous data. Prices of pork and chicken along with various consumer price indices are included in the data set. These adjustment factors along with the pork and chicken prices that are utilized in this study are the same ones that Jensen and Schroeter used. Panel households completed questionnaires as a means of reporting demographic information including family size, ages of heads of household, educational level, employment status, occupation, race, and income level. Generally speaking, this information was available in categorical form only. For example, the children variable had categories that identified households with no children, with children in the zero to six age group, with children in the six to twelve age group, with children in the twelve to eighteen age group, with children in the zero to six age group and in the six to twelve age group, with children in the zero to six age group and in the twelve to eighteen age group, with children in the six to twelve age group and in the twelve to eighteen age group, and with children in all three age groups.

Most demand studies use aggregate data while this experiment utilizes household specific information. An advantage of this is that inferences can be made about these specific household demographics that the beef industry can utilize to target more efficiently a specific advertising audience.

MODEL

The main statistical problem in this analysis is the frequent occurrence of zero purchases of a given beef type by a given household in a given period. Therefore, the use of a limited dependent variable model is indicated. The most common model in this situation is the Tobit Model.[5] Also, since the data set has a panel structure, the analysis needs to allow for the possibility of household effects. Therefore, a random effects model will be used to account for the variations in beef purchase behavior among households that are not explained by the regressors.[6]

The Random Effects Tobit Model is described by Maddala (1987):

$$Y_{it}^{*} = \alpha_i + \beta' x_{it} + u_{it}$$

and $\quad Y_{it} = > Y_{it}^{*}$ if $Y_{it}^{*} > 0$

$\quad Y_{it} = 0$ otherwise

for $i = 1, \ldots, I$ and $t = 1, \ldots, T$

where y_{it} is the dependent variable for the ith household and tth time period,[7]

β is a k x 1 vector of unknown parameters,

x_{it} is a k x 1 vector of known constants (explanatory variables),[8]

$\alpha_i \sim$ iid $N(0, \sigma_\alpha^2)$ accounts for the random effect of the ith household, and

$u_{it} \sim$ iid $N(0, \sigma_u^2)$ is the error term for the ith household and the tth time period.

The α_i and u_{it} are referred to as stochastic terms.

Without the random household effects, maximum likelihood estimation of the model would be straightforward. Likewise, without the limited dependent variable complication, standard generalized least squares (GLS) estimation could be used. But because these two features are combined, computational difficulties arise in estimating this model by maximum likelihood, and standard GLS is inefficient because the dependent variables do not follow a normal distribution. With independent stochastic terms, the part of the likelihood function corresponding to zero purchase observations is a product of ordinates of the cumulative distribution function (CDF) for univariate standard distributions, and the part corresponding to non-zero purchase observations is the usual normal likelihood for a linear model. The random effects structure introduces dependence among observations, so the probability of observing the samples' zero purchases for household i is:

$$\begin{aligned} &\mathrm{P}(Y_{i,t1} = 0,\ Y_{i,t2} = 0,\ \ldots,\ Y_{i,tN} = 0) \\ &= \mathrm{P}(Y^{*}_{i,t1} \leq 0,\ Y^{*}_{i,t2} \leq 0,\ \ldots,\ Y^{*}_{i,tN} \leq 0). \end{aligned} \tag{1}$$

Let $f(\alpha_i)$ be the marginal probability density function of α_i, let $Z_i = \{t\colon 1 \leq t \leq T,\ Y_{it} = 0\}$, and let $t1, t2, \ldots, tN$ be time periods in which $Y_{it} = 0$. If we condition on α_i, then (1) becomes

$$\begin{aligned} &\int \mathrm{P}(Y^{*}_{i,t1} \leq 0,\ Y^{*}_{i,t2} \leq 0,\ \ldots,\ Y^{*}_{i,tN} \leq 0 \mid \alpha_i) f(\alpha_i) \\ &= \int_{-\infty}^{\infty} \prod_{t \in Z_i} P\left[\alpha_i + \beta' x_{it} + u_{it} \leq 0 \middle| \alpha_i\right] f(\alpha_i)\, d\alpha_i. \end{aligned} \tag{2}$$

Now the probability of observing zero purchases for all households is computed by multiplying I expressions of the form of equation (2) because the households' purchase activities are assumed to be independent. The result is:

$$\prod_{i=1}^{I} \int_{-\infty}^{\infty} \left(\prod_{t \in Z_i} F_{it} \right) \frac{1}{\sigma_\alpha} \phi\left(\frac{\alpha_i}{\sigma_\alpha} \right) d\alpha_i \tag{3}$$

where $F_{it} = \Phi\left(\frac{-\beta' x_{it} - \alpha_i}{\sigma_u} \right)$. Equation (3) is computationally difficult to evaluate. The maximum likelihood estimation procedure would require optimizing the likelihood function numerically, which would involve repeated evaluation of (3).

Because of these difficulties, a Bayesian approach to estimation of the parameters will be used as an alternative to maximum likelihood. Bayesian inference involves prior information on the parameters and updating that information through the likelihood function to form a posterior distribution. The posterior distribution is based on the law of conditional probability, one version of which is

$$p(\theta \mid Y) = \frac{p(Y|\theta) \cdot p(\theta)}{\int p(Y|\theta) \cdot p(\theta) d\theta} \propto p(Y|\theta) \cdot p(\theta),$$

where θ is the vector of parameters, Y is the vector of data, and $p(\cdot)$ represents a generic probability density function. The posterior distribution is represented by $p(\theta|Y)$, the likelihood function is represented by $p(Y|\theta)$, and the prior information is represented by $p(\theta)$.

Computational difficulties similar to those that arise in maximum likelihood estimation are also involved in determining the posterior distribution. Because of this, a posterior simulation technique will be used. There are many types of posterior simulators available. The one utilized in this analysis is Gibbs sampling.[9]

Gibbs sampling was first used by Geman and Geman in 1984 in image-processing models. Its statistical implications were discovered by Gelfand and Smith in 1990. The Gibbs sampler is a technique for simulating draws from a joint distribution based on the associated conditional distributions. This is useful when the joint density function is intractable, but the conditional densities are more manageable. The theory is based on elementary properties of Markov chains (Casella and George 1992).

When applied to Bayesian posterior distribution simulation, the Gibbs sampler starts with a partition of the parameter set. Let θ be a vector of parameters $(\theta_1, \theta_2, ..., \theta_n)$ and let Y represent the vector of observed data in the model. The θ_i may be subvectors or scalar elements of θ. The process starts with initial values for the parameters, i.e. $\theta^{(0)} = (\theta_1^{(0)}, \theta_2^{(0)}, ..., \theta_n^{(0)})$. Parameter values from prior information or values randomly drawn from some initial distribution may be used. Then successive draws from the conditional distributions $\theta_i \mid \theta_1, \theta_2, ..., \theta_{i-1}, \theta_{i+1}, ..., \theta_n, Y$ are obtained beginning with $i = 1$. The values from these draws replace the value of that parameter in the conditioning vector. To begin, a value for θ_1, $\theta_1^{(1)}$, is drawn from the conditional distribution of $\theta_1 \mid \theta_2^{(0)}, ..., \theta_n^{(0)}, Y$. Then $\theta_1^{(1)}$ replaces $\theta_1^{(0)}$ in the conditioning vector, and a value for θ_2, $\theta_2^{(1)}$, is drawn from the conditional distribution of $\theta_2 \mid \theta_1^{(1)}, \theta_3^{(0)}, ..., \theta_n^{(0)}, Y$. By cycling through each of the components of the θ vector, $\theta^{(1)} = (\theta_1^{(1)}, \theta_2^{(1)}, ..., \theta_n^{(1)})$ is eventually obtained. To begin the second iteration, another value of θ_1, $\theta_1^{(2)}$, is drawn, this time from the distribution of $\theta_1 \mid \theta_2^{(1)}, ..., \theta_n^{(1)}, Y$. Cycling through all components of θ produces $\theta^{(2)} = (\theta_1^{(2)}, \theta_2^{(2)}, ..., \theta_n^{(2)})$. Under general conditions (to be stated below), the $\theta^{(i)}$'s generated converge in distribution to $p(\theta|Y)$. After an appropriate convergence is achieved, $\theta^{(i)}$ is used as a part of a sample from the joint distribution of $p(\theta|Y)$. The entire process can either be repeated with a different starting point or random number seed or continued from the point of convergence to obtain additional realizations of θ for this sample.

A condition that is sufficient for convergence of the Gibbs sampler (Geweke 1995) is that for every point $\theta^* \in \Theta$ ($\Theta = \mathbb{R}^k \mathrm{X} \mathbb{R}^l \mathrm{X} (\mathbb{R}^+)^2 \mathrm{X} (\mathbb{R}^-)^{N_{zero}}$, N_{zero} is the number of zero dependent variables observed) and every $\Theta_1 \subseteq \Theta$ with the property $P[\theta \in \Theta_1|Y] > 0$, it is the case that

$$\int_{\Theta_1 \subseteq \Theta} \prod_{l=1}^{n} p(\theta_l^* | \theta_j^* (j > l), \theta_l^{(i+1)} (j < l), Y) d\theta^{(i+1)} > 0.$$

Gelfand and Smith (1990) show that the rate of convergence is a geometric rate of i, that is

$$\sup_{x \in \Theta} \left| p_{\theta^{(i)}}(x|Y) - p_\theta(x|Y) \right| \leq \kappa \rho^i$$

where $0<\rho<1$, $\kappa>0$, and Θ is as defined above. They also establish an ergodic theorem which states that

$$\lim_{i \to \infty} \frac{1}{i} \sum_{l=1}^{i} T(\theta_1^{(l)}, \ldots, \theta_n^{(l)}) = \mathrm{E}[T(\theta_1, \theta_2, \ldots, \theta_n)| \, Y]$$

for any function $T(\cdot)$ of the parameter vector θ. This result justifies the use of Gibbs sample moments as estimators of the corresponding population moments for the parameters of the posterior distribution.

There have been several studies done using the Gibbs sampler. Chib has done inference on the Tobit model (1992) and regression with autoregressive errors (1993) using a Gibbs sampler approach. Zeger and Karim (1991) used the Gibbs sampler on a Generalized Linear Model with random effects. The ideas in these studies will be combined to analyze the random effects Tobit model for fresh beef demand.

CONDITIONAL DISTRIBUTIONS OF THE PARAMETERS

The prior distributions are assumed to be normal-gamma. This choice means that the random effects $\alpha = (\alpha_1, \ldots, \alpha_I)'$ and the parameters for the regressors $\beta = (\beta_1, \ldots, \beta_k)$ have a normal distribution (conditional on the variance parameters), and the parameters for the variances $\sigma = (\sigma_\alpha^2, \sigma_u^2)$ have an inverse gamma distribution. These are the most widely used distributions for informative priors because they are natural conjugates of a normal likelihood; that is, when combined with a normal likelihood, they yield posterior distributions that are of the same form as the priors (Greene 1993).

The univariate normal distribution of X is denoted by $N(\mu, \tau^2)$ where the pdf of X is given by:

$$f(x) = \frac{1}{\tau\sqrt{2\pi}} \exp\left(-\frac{1}{2\tau^2}(x-\mu)^2\right); \ -\infty < x < \infty, \ -\infty < \mu < \infty, \text{ and } \tau > 0.$$

The mean of X is μ, and the variance of X is τ^2. If $\mu = 0$ and $\tau^2 = 1$, then X is called a standard normal variate.

The multivariate normal distribution of $X=(X_1, \ldots, X_k)$ is denoted by $MVN(\mu, \Sigma)$ where the pdf of X is given by:

$$f(x) = \frac{1}{\sqrt{(2\pi)^k |\Sigma|}} \exp\left(-\frac{1}{2}(x-\mu)'\Sigma^{-1}(x-\mu)\right); \ -\infty < x_i < \infty, \ -\infty < \mu_i < \infty, \text{ and } \Sigma$$

is positive definite. The mean vector of X is μ, and the variance-covariance matrix of X is Σ.

The inverse gamma distribution of X is denoted by $IG(\kappa,\lambda)$ where the pdf of X is given by:

$$f(x) = \frac{1}{\Gamma(\kappa)\lambda^{\kappa} x^{\kappa+1}} \exp\left(-\frac{1}{\lambda x}\right); \; x > 0,\ \kappa > 0, \text{ and } \lambda > 0$$

where $\Gamma(\cdot)$ represents the gamma function defined as

$$\Gamma(\kappa) = \int_0^{\infty} t^{\kappa-1} e^{-t} dt .$$

The mean of X is $1/[\lambda(\kappa-1)]$, $\kappa > 1$, and the variance of X is $1/[\lambda^2(\kappa-1)^2(\kappa-2)]$, $\kappa > 2$. For convenience in later computations, we reparameterize κ and λ as $(\nu-1)/2$ and $2/\nu s^2$, respectively. In problems with prior information from a previous study, ν is interpreted as the degrees of freedom in the previous study, and s^2 is interpreted as the sample variance in the previous study.

Normal-gamma priors include normal priors for random effects and the parameters for the regressors and inverse gamma distributions for the variances of the error terms. Specifically, the priors are:

$$p(\alpha \mid \sigma) \sim MVN(0, \sigma_{\alpha}^2 I)$$

where I is the identity matrix,

$$p(\beta \mid \sigma) \sim MVN(\beta_0, A^{-1})$$

where β_0 is the mean vector of the prior distribution and A^{-1} is the variance-covariance matrix of the prior distribution,

$$p(\sigma_{\alpha}^2) \sim IG\left(\frac{\nu_{\alpha}-1}{2}, \frac{2}{\nu_{\alpha} s_{\alpha}^2}\right), \text{ and}$$

$$p(\sigma_{u}^2) \sim IG\left(\frac{\nu_{u}-1}{2}, \frac{2}{\nu_{u} s_{u}^2}\right).$$

To simplify Bayesian analysis of the model, the parameter vector (β, α, σ) will be augmented with the latent values of $\alpha_i + \beta' x_{it} + u_{it}$ corresponding to the zero-valued dependent variables (Chib 1992). Define the random variables:

$$Y_{it}^{*} = \begin{cases} Y_{it} & if \quad Y_{it} > 0 \\ Z_{it} & if \quad Y_{it} = 0 \end{cases} \tag{4}$$

where Z_{it} has a truncated normal distribution from $-\infty$ to 0 with mean $\alpha_i + \beta' x_{it}$ and variance σ_u^2. To derive the conditional CDF of Z_{it}, first consider the untruncated random variable $Z_{it}^{*} \sim N(\alpha_i + \beta' x_{it}, \sigma_u^2)$. Then:

$$F(z_{it}|\alpha, \beta, \sigma, Y) = P[Z_{it} \le z_{it}|\alpha, \beta, \sigma, Y] = \frac{P[Z_{it}^{*} \le z_{it}, Z_{it}^{*} \le 0|\alpha, \beta, \sigma, Y]}{P[Z_{it}^{*} \le 0|\alpha, \beta, \sigma, Y]}$$

$$= \frac{\Phi\left(\dfrac{z_{it} - \alpha_i - \beta' x_{it}}{\sigma_u}\right)}{1 - \Phi\left(\dfrac{\alpha_i + \beta' x_{it}}{\sigma_u}\right)}$$

where Φ is the CDF for a standard normal variate. To simulate z_{it} using the inverse CDF method, the CDF of Z_{it} is set equal to U where U is a random drawing from a uniform distribution with endpoints 0 and 1.[10]

$$F(z_{it}) = U \Rightarrow \frac{\Phi\left(\dfrac{z_{it} - \alpha_i - \beta' x_{it}}{\sigma_u}\right)}{1 - \Phi\left(\dfrac{\alpha_i + \beta' x_{it}}{\sigma_u}\right)} = U$$

$$\Rightarrow \Phi\left(\frac{z_{it} - \alpha_i - \beta' x_{it}}{\sigma_u}\right) = U\left\{1 - \Phi\left(\frac{\alpha_i + \beta' x_{it}}{\sigma_u}\right)\right\}$$

$$\Rightarrow \frac{z_{it} - \alpha_i - \beta' x_{it}}{\sigma_u} = \Phi^{-1}\left[U\left\{1 - \Phi\left(\frac{\alpha_i + \beta' x_{it}}{\sigma_u}\right)\right\}\right]$$

$$\Rightarrow z_{it} - \alpha_i - \beta' x = \Phi^{-1}\left[U\left\{1 - \Phi\left(\frac{\alpha_i + \beta' x_{it}}{\sigma_u}\right)\right\}\right]\sigma_u$$

$$\Rightarrow z_{it} = \Phi^{-1}\left[U\left\{1 - \Phi\left(\frac{\alpha_i + \beta' x_{it}}{\sigma_u}\right)\right\}\right]\sigma_u + \alpha_i + \beta' x.$$

The random variable Y_{it}^* defined by (4) is now normal with mean $\alpha_i + \beta\, x_{it}$ and variance σ_u^2.

For the derivations of the conditional distributions, let Y be a vector consisting of the observations in which $Y_{it} > 0$, let z be a vector consisting of the simulated latent variables for observations in which $Y_{it} = 0$, let $Y^* = [Y_{it}^*]_{IT \times 1}$, $Y_i^* = (Y_{i1}^*, \ldots, Y_{iT}^*)$ and let $\alpha_{-i} = (\alpha_1, \ldots, \alpha_{i-1}, \alpha_{i+1}, \ldots, \alpha_I)'$.

Since the random effects (α) are independent and identically distributed, the conditional distribution of one random effect will be derived from the likelihood function and the prior information on α:

$$p(\alpha_i|\alpha_{-i}, \beta, \sigma, z, Y) \propto p(Y_i^*|\alpha_i, \beta, \sigma)\, p(\alpha_i|\beta, \sigma)\, p(\beta, \sigma)$$

$$\propto p(Y_i^*|\alpha_i, \beta, \sigma)\, p(\alpha_i|\sigma)$$

where $p(\beta,\sigma)$ has been removed as a constant of proportionality (independent of α_i), and we note that the prior density for α_i does not depend on β. Letting $\phi(\cdot)$ denote the pdf of a standard normal variate,

$$p(\alpha_i|\alpha_{-i}, \beta, \sigma, z, Y) \propto \left\{\prod_{t=1}^{T} \frac{1}{\sigma_u} \phi\left(\frac{Y_{it}^* - \alpha_i - \beta' x_{it}}{\sigma_u}\right)\right\} \frac{1}{\sigma_\alpha} \phi\left(\frac{\alpha_i}{\sigma_\alpha}\right)$$

$$\propto \exp\left\{-\left[\sum_{t=1}^{T}\left(\frac{(Y_{it}^* - \alpha_i - \beta' x_{it})^2}{2\sigma_u^2}\right) + \frac{\alpha_i^2}{2\sigma_\alpha^2}\right]\right\}$$

$$= \exp\left\{-\left[\frac{1}{2\sigma_u^2}\sum_{t=1}^{T}\left(Y_{it}^{*2} - 2Y_{it}^{*}\beta' x_{it} + \alpha_i^2 - 2\alpha_i Y_{it}^{*} + 2\alpha_i \beta' x_{it} + \beta' x_{it} x_{it}^{'} \beta\right) + \frac{\alpha_i^2}{2\sigma_\alpha^2}\right]\right\}.$$

Again removing factors that are independent of α_i as a proportionality constant, we have

$$p(\alpha_i|\alpha_{-i}, \beta, \sigma, z, Y) \propto \exp\left\{-\left[\frac{T\alpha_i^2}{2\sigma_u^2} + \frac{\alpha_i^2}{2\sigma_\alpha^2} - \frac{2\alpha_i \sum_{t=1}^{T} Y_{it}^{*}}{2\sigma_u^2} + \frac{2\alpha_i \beta' \sum_{t=1}^{T} x_{it}}{2\sigma_u^2}\right]\right\}.$$

To complete the square in the exponent, define

$$\tau^2 = \left(\frac{T}{\sigma_u^2} + \frac{1}{\sigma_\alpha^2}\right)^{-1} \text{ and } \mu = \left(\frac{\sum_{t=1}^{T} Y_{it}^{*}}{\sigma_u^2} - \frac{\beta' \sum_{t=1}^{T} x_{it}}{\sigma_u^2}\right)\tau^2$$

and introduce the proportionality constant $\exp\left(-\frac{\mu^2}{2\tau^2}\right)$ to obtain

$$p(\alpha_i|\alpha_{-i}, \beta, \sigma, z, Y) \propto \exp\left\{\frac{-1}{2\tau^2}\alpha_i^2 + \frac{2\alpha_i \mu}{2\tau^2} - \frac{\mu^2}{2\tau^2}\right\}$$

$$= \exp\left\{-\frac{1}{2\tau^2}(\alpha_i - \mu)^2\right\} \propto N(\mu, \tau^2).$$

Hence the conditional distribution of α_i is normal with mean μ and variance τ^2.

The conditional distribution of the regression parameters (β) is derived from the likelihood function and prior information on β:

$$p(\beta|\alpha, \sigma, z, Y) \propto p(Y^*|\alpha, \beta, \sigma)p(\alpha|\beta, \sigma)p(\beta|\sigma)p(\sigma)$$

$$\propto p(Y^*|\alpha, \beta, \sigma)p(\beta|\sigma)$$

where $p(\alpha|\beta,\sigma)$ and $p(\sigma)$ are removed as part of the proportionality constant because they are independent of β, noting that $p(\alpha|\beta,\sigma)$ does not depend on β. Now let $d = [d_{it}]_{N \times 1} = [Y_{it}^{*} - \alpha_i]_{N \times 1}$, where $N=IT$. Then

$$p(\beta\,|\alpha,\sigma,z,Y) \propto \exp\left\{-\frac{1}{2}\left[(d - X\beta)'\sigma_u^{-2}I(d - X\beta) + (\beta - \beta_0)'A(\beta - \beta_0)\right]\right\}$$

$$= \exp\left\{-\frac{1}{2}(d'\sigma_u^{-2}d - d'\sigma_u^{-2}X\beta - \beta'X'\sigma_u^{-2}d + \beta'X'\sigma_u^{-2}X\beta)\right\}$$

$$\times \exp\left\{-\frac{1}{2}(\beta'A\beta - \beta'A\beta_0 - \beta_0'A\beta + \beta_0'A\beta_0)\right\}.$$

Again removing factors independent of β as a proportionality constant and to complete the square in the exponent, defining

$$\Sigma = \left(\frac{X'X}{\sigma_u^2} + A\right)^{-1} \text{ and } \beta^{*} = \Sigma\left(\frac{X'd}{\sigma_u^2} + A\beta_0\right),$$

we have

$$p(\beta\,|\alpha,\sigma,z,Y) \propto \exp\left\{-\frac{1}{2}[\beta'\Sigma^{-1}\beta - \beta^{*}{}'\Sigma^{-1}\beta - \beta'\Sigma^{-1}\beta^{*}]\right\}.$$

We introduce the proportionality constant $\exp\left\{-\frac{1}{2}\beta^{*}{}'\Sigma^{-1}\beta^{*}\right\}$ to obtain

$$p(\beta\,|\alpha,\sigma,z,Y) \propto \exp\left\{-\frac{1}{2}[\beta'\Sigma^{-1}\beta - \beta^{*}{}'\Sigma^{-1}\beta - \beta'\Sigma^{-1}\beta^{*} + \beta^{*}{}'\Sigma^{-1}\beta^{*}]\right\}$$

$$\propto \exp\left\{-\frac{1}{2}(\beta - \beta^{*})'\Sigma^{-1}(\beta - \beta^{*})\right\} \propto MVN(\beta^{*},\Sigma).$$

Hence the conditional distribution of β is multivariate normal with mean vector β^{*} and variance-covariance matrix Σ.

The conditional distributions of the variances (σ) are derived from the likelihood function and prior information on α and σ:

$$p(\sigma|,\alpha,\beta,z,Y) \propto p(Y^*|\alpha,\beta,\sigma)p(\alpha|\beta,\sigma)p(\beta|\sigma)p(\sigma)$$

$$\propto p(Y^*|\alpha,\beta,\sigma)p(\alpha|\sigma)p(\sigma)$$

where $p(\beta|\sigma)$ is independent of σ; therefore, it is removed as a proportionality constant.

Now

$$p(\sigma|\alpha,\beta,z,Y) \propto \left\{\prod_{t=1}^{T}\prod_{i=1}^{I}\frac{1}{\sigma_u}\phi\left(\frac{Y_{it}^* - \alpha_i - \beta' x_{it}}{\sigma_u}\right)\right\}\left\{\prod_{i=1}^{I}\frac{1}{\sigma_\alpha}\phi\left(\frac{\alpha_i}{\sigma_\alpha}\right)\right\}$$

$$\times\left(\frac{1}{\sigma_\alpha^{\nu_\alpha+1}}\exp\left\{-\frac{\nu_\alpha s_\alpha^2}{2\sigma_\alpha^2}\right\}\right)\left(\frac{1}{\sigma_u^{\nu_u+1}}\left\{-\frac{\nu_u s_u^2}{2\sigma_u^2}\right\}\right)$$

$$=\left(\prod_{t=1}^{T}\prod_{i=1}^{I}\frac{1}{\sigma_u}\phi\left[\frac{Y_{it}^* - \alpha_i - \beta' x_{it}}{\sigma_u}\right]\right)\left(\frac{1}{\sigma_u^{\nu_u+1}}\exp\left\{-\frac{\nu_u s_u^2}{2\sigma_u^2}\right\}\right)$$

$$\times\left(\prod_{i=1}^{I}\frac{1}{\sigma_\alpha}\phi\left[\frac{\alpha_i}{\sigma_\alpha}\right]\right)\left(\frac{1}{\sigma_\alpha^{\nu_\alpha+1}}\exp\left\{-\frac{\nu_\alpha s_\alpha^2}{2\sigma_\alpha^2}\right\}\right) \propto p(\sigma_u^2|\alpha,\beta,z,Y)\times p(\sigma_\alpha^2|\alpha,\beta,z,Y).$$

Thus, $\sigma_u^2|\alpha, \beta, z, Y$ and $\sigma_\alpha^2|\alpha, \beta, z, Y$ are stochastically independent.

First the conditional distribution of the error variance and then the conditional distribution of the random effects variance will be derived:

$$p(\sigma_u^2|\alpha,\beta,z,Y) \propto \frac{1}{\sigma_u^{IT+\nu_u+1}}\exp\left\{-\frac{1}{2\sigma_u^2}\left[\sum_{i=1}^{I}\sum_{t=1}^{T}(Y_{it}^* - \alpha_i - \beta' x_{it})^2 + \nu_u s_u^2\right]\right\}$$

$$\propto IG\left(\frac{IT+\nu_u-1}{2}, \frac{2}{\sum_{t=1}^{T}\sum_{i=1}^{I}(Y_{it}^* - \alpha_i - \beta' x_{it})^2 + \nu_u s_u^2}\right), \text{ and}$$

$$p(\sigma_{\alpha}^{2}|\alpha\ ,\beta\ ,z,Y)\propto\frac{1}{\sigma_{\alpha}^{I+\nu_{\alpha}+1}}\exp\left\{-\frac{1}{2\sigma_{\alpha}^{2}}\left[\sum_{i=1}^{I}(\alpha_{i})^{2}+\nu_{\alpha}s_{\alpha}^{2}\right]\right\}$$

$$\propto IG\left(\frac{I+\nu_{\alpha}-1}{2},\frac{2}{\sum_{i=1}^{I}\alpha_{i}^{2}+\nu_{\alpha}s_{\alpha}^{2}}\right).$$

Hence the conditional distributions of σ_α^2 and σ_u^2 are inverse gamma with the above parameters.

To do the Gibbs sampling, a FORTRAN program was written using subroutines from the NAG library.[11] This program is in the appendix. The basic algorithm of the program is as follows:

Step 1: Input data for the dependent variable (household purchases of steaks, roasts, or ground beef) and the explanatory variables (beef and substitute prices, household demographics, etc.)

Step 2: Set the length of the burn-in period (nburn): The number of Gibbs loops that will be executed before accumulation of the sample begins.

Step 3: Set the total number of Gibbs loops to be executed (ngibbs).

Step 4: Set the frequency with which loops between nburn and ngibbs will be used to augment the sample (k). [12].

Step 5: Set values for parameters of the prior distributions on β, α, and σ.

Step 6: Set initial values for β, α, and σ: $\beta^{(0)}$, $\alpha^{(0)}$, $\sigma^{(0)}$.

Do i = 1 to ngibbs:

Step 7: Draw values for latent beef purchases, $z^{(i)}|\alpha^{(i-1)}$, $\beta^{(i-1)}$, $\sigma^{(i-1)}$, Y.

Step 8: Draw values for $\alpha^{(i)}|z^{(i)}, \beta^{(i-1)}, \sigma^{(i-1)}, Y$.

Step 9: Draw values for $\beta^{(i)}|\ z^{(i)}, \alpha^{(i)}, \ \sigma^{(i-1)}, Y$.

Step 10: Draw values for $\sigma^{(i)}|\ z^{(i)}, \alpha^{(i)}, \beta^{(i)}, Y$.

Step 11: If $i >$ nburn and if i/k is an integer, then output parameter draw to a file; otherwise, continue.

End Do Loop.

Because no prior information is available, prior means for β_i are set to zero[13] and prior variances for β are set to be very high.[14] This makes the prior almost diffuse. For the prior degrees of freedom and prior variances on $p(\sigma)$, the degrees of freedom are set equal to one for both ν_α and ν_u, representing no prior information, and s_α^2 and s_u^2 are set equal to one-half to represent no prior information about the stochastic term variances. A low value of s_i^2 ($i = \alpha, u$) results in a lower value for the prior expectation of σ_i^2.

MODEL VARIABLES

The variables in these models are defined the same as in Jensen and Schroeter (1992) with a few changes. The households that are modeled are those with both male and female heads. In household decision making, concerns include efficient use of market goods, time, and human capital (Deaton and Muelbauer 1980). Single parent households have many different decision making concerns that a two-parent household does not face. Two-parent households usually have more time to prepare a meal, and they have more human capital to supply which usually results in more income, while many times single-parent households will make more efficient use of market goods because of a more limited income. Because of these differences, only two-parent households' observations are included in the model. The total number of these households used in the model is 1450.

Dependent Variables

There will be three dependent variables modeled. They are household purchase quantities of steak, roast, and ground beef. Each will be modeled separately, on the basis of the assumption that each demand equation's error components are uncorrelated with the error components of the other demand equations. The possibility of cross-equation correlation in the error terms will be discussed later.

In estimating food demand, the household purchase quantities need to be standardized according to household size and composition (Jensen and Schroeter 1992). Tedford, Capps and Havlicek (TCH, 1986) used concepts from the fields of child and adult nutrition to develop a scale that gauges the relative consumption needs of individuals of different ages and sexes. A prime age adult male is assigned a weight of one and lower weights are assigned to individuals of other age-sex combinations. Because of the categorical nature of the Grand Junction data set, exact inferences about household composition are not possible. Therefore, for each category which can be identified, simple averages of TCH factors are computed. For example, the data reveal only the households' number of males in the 18-29 year age range, not the specific ages of household members in that category. So each is assigned the sample average of TCH factors for males aged 18, 19, 20, ..., and 29; 0.997589. Similarly, the sexes of children and certain adult members of the household could not be inferred. These "unisex" household members were assigned weights that were the average for male and female TCH factors for the corresponding ages. The resulting household member consumption weights for each category are given in Table 2.

The sum of the consumption weights for each household is the household's number of "adult-male equivalents" (AME). The size of each household is then measured by its number of "standard persons," defined as the household's number of adult male equivalents divided by the panel-wide average of adult male equivalents per capita.[15] The standard person measure provides the basis for adjusting purchase quantities for household size and composition.

The data also need to be adjusted because of seasonality of demand. The adjustment factors are those used in Jensen and Schroeter and are based on estimates of

Table 2. TCH Factors

Gender	Age Group	Factor
Male	18-29	0.997589
	30-34	0.991402
	35-44	0.989387
	45-54	0.972649
	55-64	0.955841
	65-81	0.850412
Female	18-29	0.766623
	30-34	0.834053
	35-44	0.855517
	45-54	0.812787
	55-64	0.769212
	65-81	0.724403
Unisex[a]	0-5	0.464223
	6-11	0.679158
	12-17	0.840754
	18-81	0.867258

[a]Unisex is for the categories in which information on gender is not available.

"a single-equation, national, monthly beef demand model in which the dependent variable is the logarithm of monthly U.S. beef disappearance per day" (Jensen and Schroeter 1992), and among the explanatory variables are monthly dummy variables. The coefficients of each monthly dummy variable represents the percentage departure between beef consumption in the given month and in a "standard" month, other things equal. These coefficients provided the factors used to seasonally adjust the household quantities from the Grand Junction experiment.

In the end, the dependent variables were taken to be the seasonally adjusted household purchase quantities (of steak, roasts, or ground beef) per standard person. These and other variables are defined in Table 3. Table 4 provides summary statistics.

Table 3. Definition of Variables

$SQPC_{it}$	=	seasonally adjusted, standardized steak purchases of household i in period t (pounds per standard person per four week period).
$RQPC_{it}$	=	seasonally adjusted, standardized roast purchases of household i in period t (pounds per standard person per four week period).
$GQPC_{it}$	=	seasonally adjusted, standardized ground beef purchases of household i in period t (pounds per standard person per four week period).
SPR_t	=	quantity weighted average of prices paid for steak by all panel members in period t, adjusted by the consumer price index of prices for food at home for all urban consumers in cities in the size class of Grand Junction in the western region of the U.S. (period 23-cents per pound).
RPR_t	=	quantity weighted average of prices paid for roasts by all panel members in period t, adjusted as in the definition of SPR (period 23-cents per pound).
GPR_t	=	quantity weighted average of prices for ground beef by all panel members in period t, adjusted as in the definition of SPR (period 23-cents per pound).
PPR_t	=	price of center-cut, bone-in-pork chops in the western region of the U.S. in period t, adjusted as in the definition of SPR (period 23-cents per pound).
CPR_t	=	price of fresh whole chicken in the western region of the U.S. in period t, adjusted as in the definition of SPR (period 23-cents per pound).
MSE_i	=	number of years of schooling of the male head of household i, if he is employed (equal to zero if he is not employed).
$MSUE_i$	=	number of years of schooling of the male head of household i, if he is not employed (equal to zero if he is employed).
FSE_i	=	number of years of schooling of the female head of household i, if she is employed (equal to zero if she is not employed).
$FSUE_i$	=	number of years of schooling of the female head of household i, if she is not employed (equal to zero if she is employed).
$CH1_i$	=	number of children in household i in the age group zero to six years.
$CH2_i$	=	number of children in household i in the age group six to twelve years.
$CH3_i$	=	number of children in household i in the age group twelve to eighteen years.
$SIZE_i$	=	number of standard persons in household i expressed as a deviation from the panel-wide average
MHM_i	=	a dummy variable equal to 1 if male head of household i is a full time "homemaker".
FHM_i	=	a dummy variable equal to 1 if female head of household i is a full time "homemaker".
MHA_i	=	age, in years, of the male head of household i.
FHA_i	=	age, in years, of the female head of household i.
NW_i	=	a dummy variable equal to 1 if household i is of a non-white race or is non-Hispanic.
$HISP_i$	=	a dummy variable equal to 1 if household i is Hispanic.
BAP_i	=	a dummy variable equal to 1 if household i is in the base ad panel.

Table 3. (continued)

HAP_i	=	a dummy variable equal to 1 if household i is in the heavy ad panel.
OWN_i	=	a dummy variable equal to 1 if household i owns their place of residence.
$DISH_i$	=	a dummy variable equal to 1 if household i owns a dishwasher.
$PHS1_t$	=	a dummy variable equal to 1 if period t is in phase 1 of the advertising test.
$PHS2_t$	=	a dummy variable equal to 1 if period t is in phase 2 of the advertising test.
$FEAT_{it}$	=	total beef purchases made on feature-priced items for household i in period t.
$FEXP_{it}$	=	standardized identification card expenditures of household i in period t, adjusted as in the definition of SPR (period 23-cents per standard person per four-week period).
ADV_{it}	=	a weighted average of current and past test advertising exposure levels for household i in period t (GRPs).

price is simply determined by dividing total expenditures by all panel members in the tth time period by the total purchase quantities by all panel members in the tth time period. This is done for all three types of fresh beef (steaks, roasts, and ground beef).

For the substitute (pork and chicken) prices, there is no information available from the scanner data, so data from secondary sources are used. Prices of center-cut, bone-in pork chops and prices of fresh whole chickens are used for substitute prices. The beef and substitute prices are adjusted by the consumer price index of prices for food at home for all urban consumers in cities in the size class of Grand Junction in the western region of the United States.

Household Demographics

Many demographic characteristics have been shown in past studies to have a significant effect on demand for all types of beef. Some of the main determinants are: household size, urbanization, ethnic background, region, tenancy (whether the household

Table 4. Variable summary statistics

Variable	Number of Observations	Mean	Standard deviation
Quantitative variables that vary over time and across households:			
$SQPC_{it}$	33,350	0.5082	1.110
$RQPC_{it}$	33,350	0.4250	1.041
$GQPC_{it}$	33,350	1.150	3.052
$FEAT_{it}$	33,350	302.50	659.5
$FEXP_{it}$	33,350	7692.2	4202.6
Quantitative variables that vary across households only:			
$CH1_i$	1,450	0.1669	0.4755
$CH2_i$	1,450	0.2834	0.6502
$CH3_i$	1,450	0.3779	.08084
$SIZE_i$	1,450	0.0000	1.115
MHA_i	1,450	52.47	14.24
FHA_i	1,450	50.51	14.07
MSE_i	954[a]	13.18	2.127
$MSUE_i$	496[a]	12.17	2.753
FSE_i	750[a]	13.05	1.751
$FSUE_i$	700[a]	12.35	2.193
Quantitative variables that vary over time only:			
SPR_t	23	275.70	21.997
RPR_t	23	185.01	12.323
GPR_t	23	133.46	6.115
PPR_t	23	292.38	16.892
CPR_t	23	89.78	5.606
Qualitative variables for household characteristics			
MHM_i	496[b]		
FHM_i	700[b]		
NW_i	9[b]		
$HISP_i$	52[b]		
BAP_i	419[b]		
HAP_i	627[b]		
OWN_i	1301[b]		
$DISH_i$	1120[b]		

[a]In these cases, the figure represents the number of households for which the variable's value is non-zero. For example, out of the 1,450 households, 954 of the males heads were employed.
[b]In these cases, the figure represents the number of households for which the variables' value is one.

owns or rents), food planner (person in the household that plans the food purchases and the meals), availability of health information, extent of away-from-home food consumption, and employment status of female head of household. Other demographic variables have been shown to have significant effects on at least one type, but not all types of beef (Heien and Pompelli 1988, Gao and Spreen 1994).

For an employed individual, an increase in the wage rate leads to a reduction in time devoted to home production activities. Because preparing meals is one such activity, an increased wage rate should reduce demand for fresh beef. These households not only decrease purchases of food to consume at home, they also purchase more foods to consume at home that have already been prepared. For someone who is unemployed, a wage increase has no marginal effect on production time at home. Because of this, the wage rate effects of unemployed adults should be included separately from those of employed adults in the model.[16]

Data for wages are not included in the data set; therefore, education level is used as a proxy for both employed and unemployed individuals. Years of schooling may also have a negative effect on fresh beef demand because more education might tend to make individuals more aware of health concerns that warn them not to consume red meat. In this case, education should have a negative effect on fresh beef demand, with the effect being stronger for employed heads of households for whom "education/health awareness" effect would be reinforced by the reduced-time-spent-in-home-production-activities effect.

Four variables are included in the model to serve as proxies for wage rates: MSE, MSUE, FSE, and FSUE which represent number of years of schooling for employed males, unemployed males, employed females, and unemployed females, respectively.

Also, the number of children in a household may have an effect on fresh beef demand. Three variables account for this effect: CH1, CH2, and CH3 which represent the number of children in age group zero to six years, in age group six to twelve years, and in age group twelve to eighteen years, respectively. The impact that children have on fresh beef demand can be either positive or negative. Either home child care and meal preparation are complementary activities or competing activities. Having older children who are able to assist in the meal preparation is an example of them being complementary activities. Having more children, especially young children, results in spending more time in child care activities, which makes home child care and meal preparation competing activities. If they are complementary, then fresh beef demand per standard person should increase. On the other hand, if they are competing with each other for the homemaker's time, a decrease is expected. Therefore, it is expected that if a household's children are older then the standardized demand for fresh beef is higher. Also, children may have age-specific preferences. For example, steaks may not appeal to young children as they may to older children.

Two "homemaker" variables are also included in the model: MHM and FHM which represent male homemaker and female homemaker, respectively. MHM is a dummy variable that is equal to one if the male head of household is unemployed, and zero otherwise. FHM is defined in the same way except that it is for female heads of household. This is included to reflect household preferences and market opportunities (Jensen and Schroeter 1992).

The ages of the heads of household could have an effect for several reasons. An older person may have more health concerns, reducing fresh beef demand. On the other hand, an older person may hold more traditional dietary attitudes and preferences that cause him or her to demand more beef or to prefer one type of fresh beef over another. Therefore, variables reflecting age for both male (MHA) and female (FHA) heads of household are included.

The SIZE variable defined as the household's number of standard persons as a deviation from its panel-wide average is also included. For meal preparation, the time cost increases less rapidly (if at all) than proportionally with the size of the beef product being prepared resulting in economies of scale. This could cause demand per standard person to increase as the number of standard persons increase. This suggests that household size should have a positive effect on the fresh beef demand per standard person.

Two ethnic variables are included to help explain differences in tastes. One of them is NW, which is a dummy variable equal to one if the household is of a non-white race or is not Hispanic, and zero otherwise. The other is HISP, which is a dummy variable equal to one if the household is Hispanic, and zero otherwise. Two other dummy variables included are OWN, which is equal to one if the household owns their place of residence, and DISH, which is equal to one if the household owns a dishwasher. The OWN variable is included because past studies have concluded that it has a significant effect on fresh beef demand. Many households that own a home also have higher income than those that do not which would increase demand for fresh beef. The DISH variable is included because households with dishwashers would have less clean up time and thus have more time for food preparation.

Store Featuring

Featured items, items that are promoted in local print advertising or in-store displays, were responsible for 25% of the beef expenditures in the Grand Junction experiment. Because such promotions are simply another form of advertising, an increase in featuring activity should increase demand for fresh beef. A variable on featuring items, FEAT, was included with the data set. It is the total expenditures on beef of "featured" items by household *i* in period *t*. A discussion on this variable and the effect of featuring items is included in the possible extensions chapter.

Income

Panel households reported income in categorical form only, so Jensen and Schroeter suggest using an income proxy that measures total expenditure on food for at-home consumption. They base this on the assumption that the household utility function is weakly separable in food items for at-home consumption and all other goods so that the demand for beef can be thought of as obtaining from maximization of a food consumption subutility subject to given food prices and given total expenditure on food.

More income, or specifically more total expenditures on food, should stimulate fresh beef demand. The variable reflecting this is called FEXP.

Advertising

The explanatory variable representing advertising's impact needs to take into account the lagged and cumulative effects of exposure. There are several approaches to doing this. The one used in this analysis was also used by Jensen and Schroeter: advertising's effect is represented by a 12-month, second-order polynomial distributed lag in advertising intensities. The lag weights are fixed and set to the specific values used by Ward and Dixon (1989) in their analysis of advertising's effect on the consumption of milk. This leads to an advertising variable defined as

$$ADV_{it} = \sum_{j=0}^{11} w_j^* GRP_{i,t-j}$$

where GRP_{it} is the number of gross rating points of advertising exposure received by household i in time period t. The w_j^*'s are the Ward and Dixon weights rescaled to sum to one.[17] The main goal of the Beef Promotion and Research Act was to stimulate beef demand; therefore, an increase in advertising exposure should lead to an increase in beef demand.

Finally, dummy variables are included to control for differences across the panels and time-periods that are not accounted for by the effects of advertising or by household demographics. They are BAP, HAP, PHS1, and PHS2: dummy variables identifying the base ad panel, heavy ad panel, phase one of advertising test, and phase two of advertising test, respectively.

CONVERGENCE RESULTS

In deciding which draws for the Gibbs sampling algorithm are going to be used in compiling the sample from the posterior distribution, there are two main issues to consider. The first is how many Gibbs loops should be undertaken before any draws are used in the sample; i.e., when has the Gibbs sampler converged? The period of draws before convergence is called the burn-in period. The second issue is whether every draw after the burn-in period should be used or just every *k*th draw.

The first issue is addressed by running the Gibbs sampler and doing a time-series plot of the parameters. If there appears to be no discernible trend in the plot of the data, then it can be assumed to have converged. A conservative burn-in period should be selected to reduce the chance of using values sampled before convergence has actually occurred.

Figure 1 plots sampled values of four parameters in the steak model as a time series. They are the own-price coefficient, the advertising coefficient, the random effects variance,[18] and the random effect for household one. They are illustrated as part of the "tests" for convergence. The main thing to look for in these plots is the point at which values settle down into a stable pattern that no longer changes through time. Each of them appears to converge early. This result is typical of the rest of the parameter plots not shown here. The burn-in period was set at 500. This value is more conservative than appears to be indicated by the convergence plots, but since there is enough information

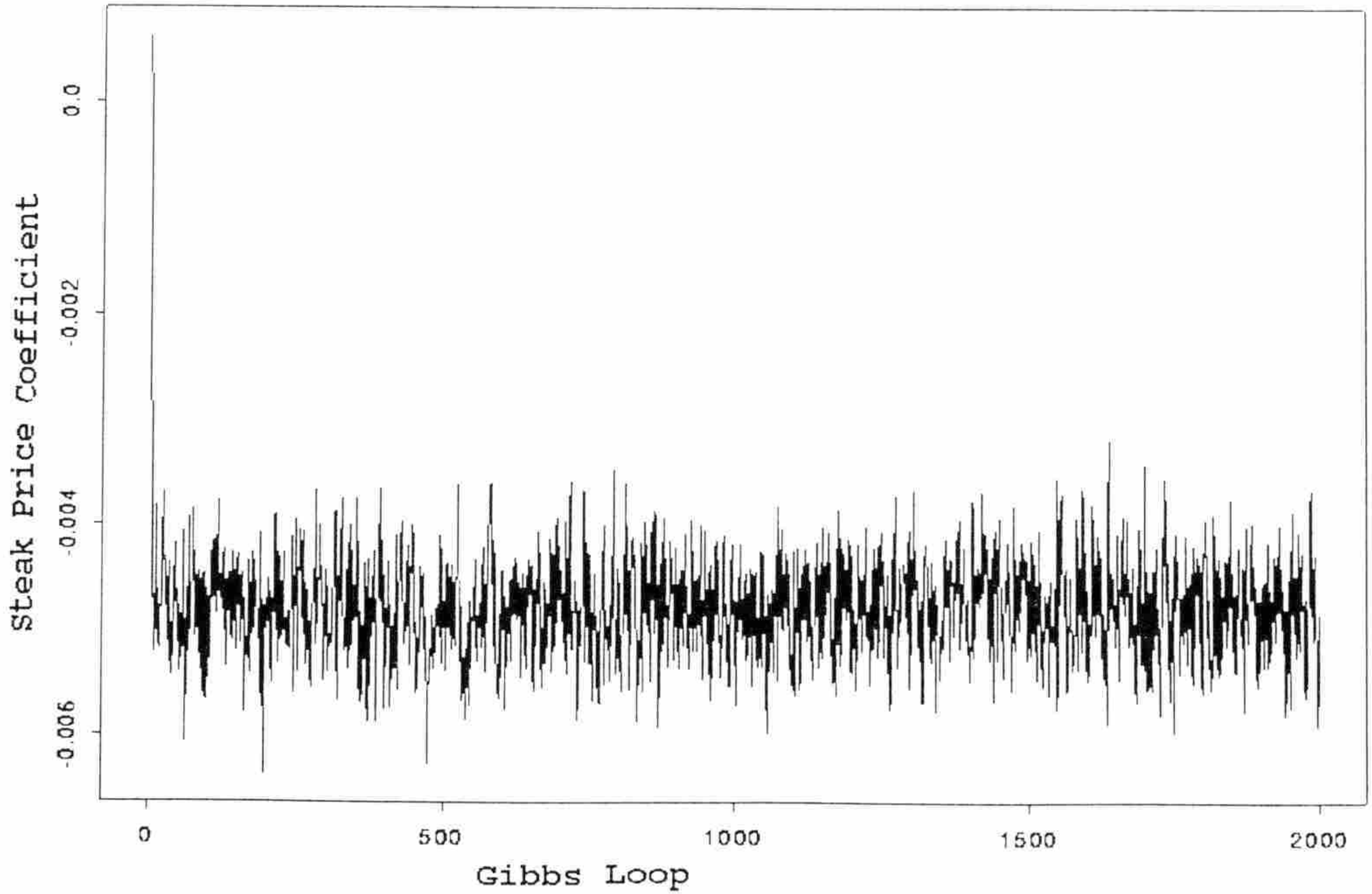

Figure 1. Time series plot of steak price effect on steak demand

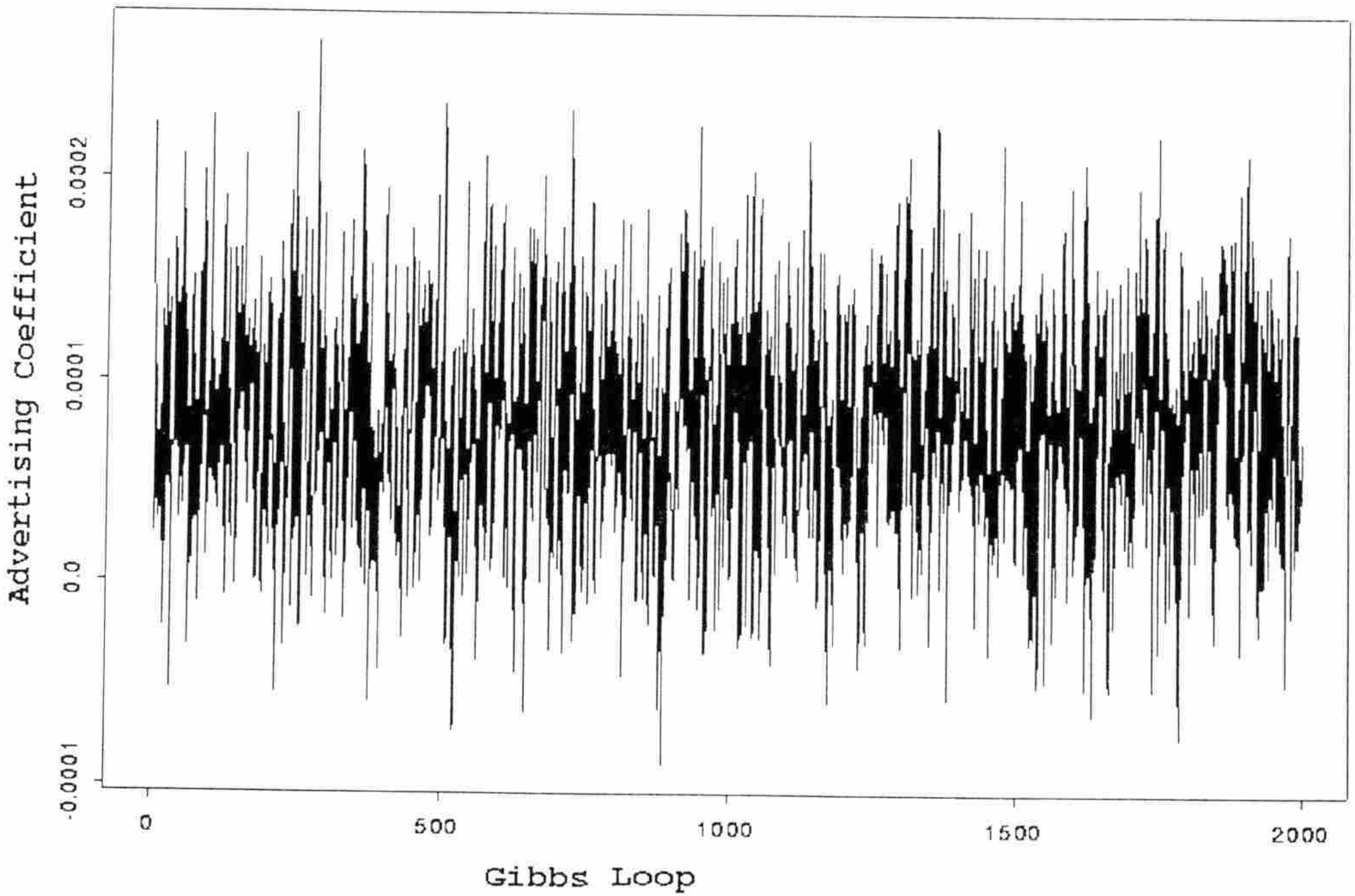

Figure 1. (continued) Time series plot of advertising effect on steak demand

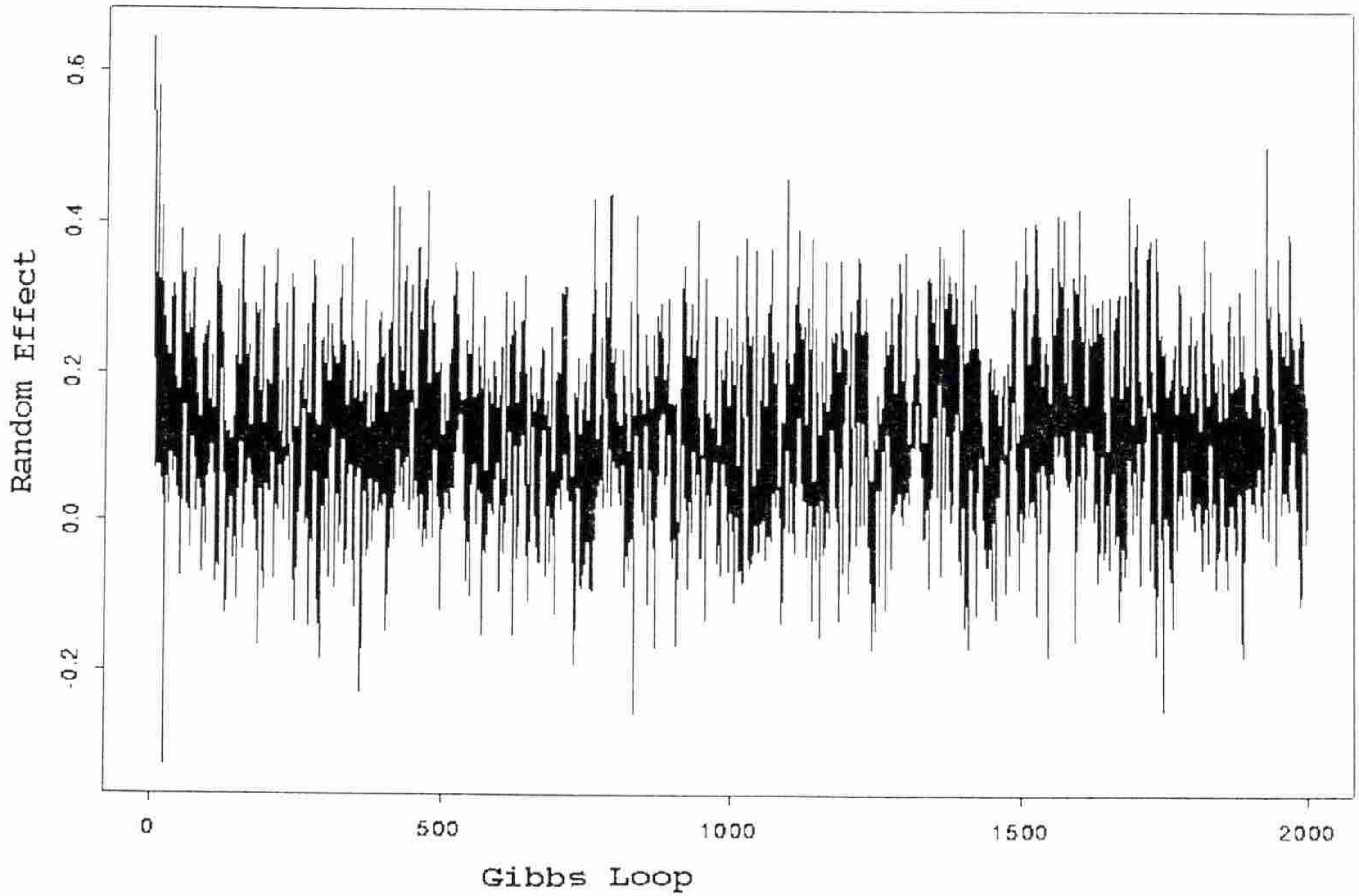

Figure 1. (continued) Time series plot of random effect for household one on steak demand

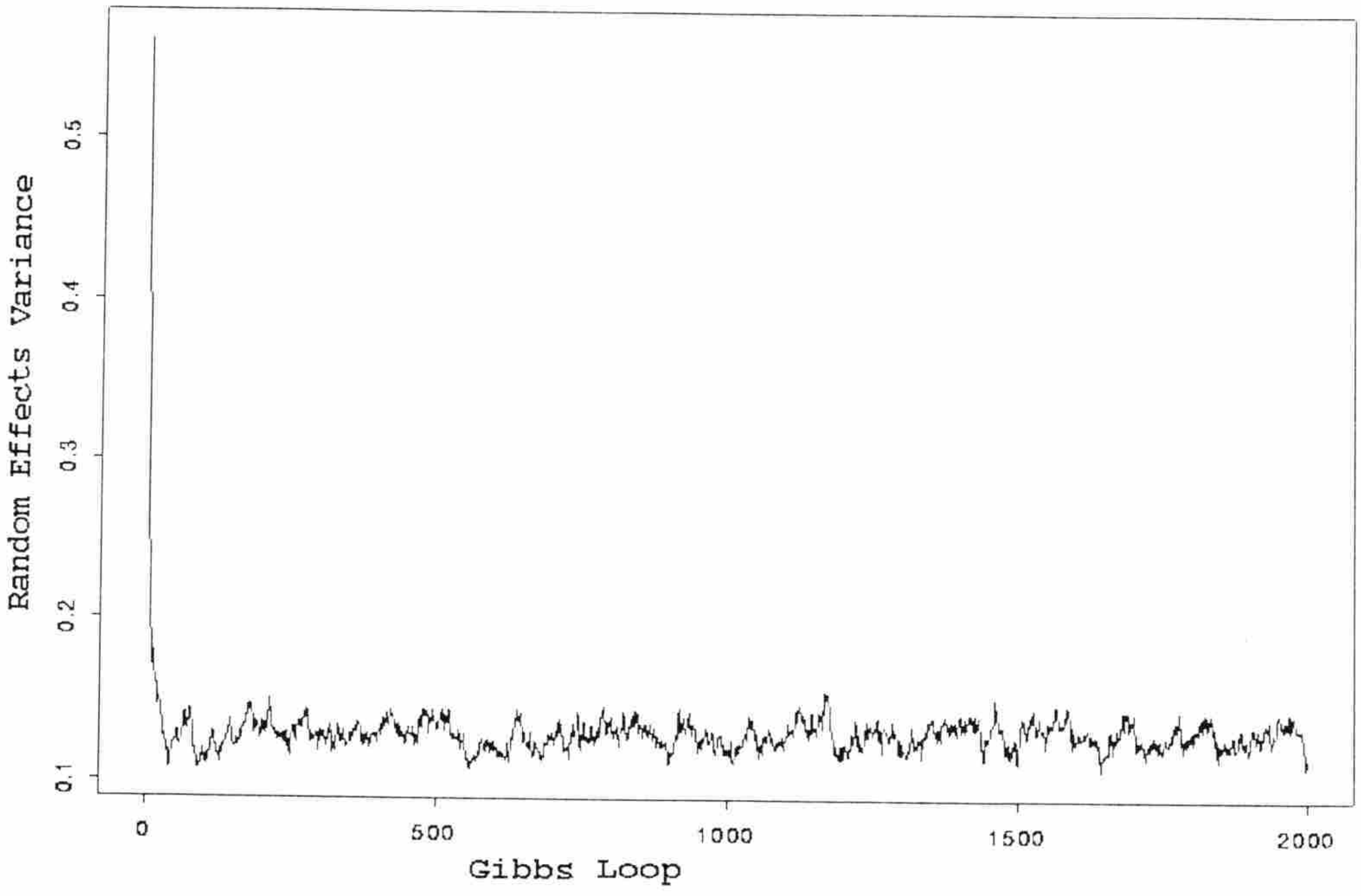

Figure 1. (continued) Time series plot of random effects variance on steak demand

available. a long burn-in period reduces the chance of sampling values before convergence.

Another test for convergence can be done by analysis of variance. Gelman, et. al., (1995) suggested comparing the between sequence variation and the within-sequence variation. As the sample size becomes large, the between sequence variation should approach zero. They are computed for each parameter of interest. Each model was simulated five times with 150 draws available for each sample. For the analysis of variance, the draws are labeled θ_{ij}, which represents the ith draw available from the jth sequence simulated. Obviously, from this sample $i = 1, \ldots, 150$ and $j = 1, \ldots, 5$. Let B and W be the between- and within-sequence variation, respectively. Then

$$B = \frac{150}{5-1}\sum_{j=1}^{5}\left(\bar{\theta}_{\cdot j} - \bar{\theta}_{\cdot\cdot}\right)^2, \text{ and } W = \frac{1}{5}\sum_{j=1}^{5} s_j^2, \text{ where } \bar{\ }_{\cdot j} = \frac{1}{150}\sum_{i=1}^{150}\theta_{ij},\ \bar{\ }_{\cdot\cdot} = \frac{1}{5}\sum_{j=1}^{5}\bar{\theta}_{\cdot j}, \text{ and}$$

$$s_j^2 = \frac{1}{150-1}\sum_{i=1}^{150}\left(\theta_{ij} - \bar{\theta}_{\cdot j}\right)^2.$$

The estimated marginal posterior variance of $\theta|Y$ is defined by:

$\mathrm{Var}^*(\theta|Y) = \frac{n-1}{n}W + \frac{1}{n}B$ where in this case $n = 150$. As $n \to \infty$, the estimated variance approaches W. Gelman, et. al., computed the values of

$$\sqrt{\hat{R}} = \sqrt{\frac{\mathrm{Var}^*(\theta\,|\,Y)}{W}}$$

which declines to 1 as $n \to \infty$. These computed values for the regressors, stochastic term variances, and the first five random effects are shown in Table 5. Gelman, et. al., suggest

Table 5. Estimates of $\sqrt{\hat{R}}$

Parameters	Steak Model	Roast Model	Ground Beef Model
β_i on:			
SPR	1.0043	1.0028	1.0004
RPR	1.0066	1.0076	1.0017
GPR	1.0050	0.9999	1.0008
PPR	1.0052	1.0007	1.0013
CPR	1.0066	1.0037	1.0000
MSE	1.0015	1.0048	1.0016
MSUE	1.0014	1.0049	1.0011
FSE	1.0015	1.0047	1.0023
FSUE	1.0027	1.0047	1.0010
CH1	1.0020	1.0032	1.0018
CH2	1.0027	1.0035	1.0057
CH3	1.0010	1.0004	1.0045
SIZE	1.0022	1.0020	1.0125
MHM	1.0035	1.0039	1.0017
FHM	1.0005	1.0002	1.0012
MHA	1.0023	1.0036	1.0013
FHA	1.0009	1.0048	1.0014
NW	0.9999	1.0013	1.0014
HISP	1.0131	1.0070	1.0062
BAP	1.0002	1.0007	1.0028
HAP	1.0023	1.0022	1.0010
OWN	1.0066	1.0039	1.0007
DISH	0.9998	1.0004	1.0047
PHS1	1.0005	1.0009	1.0043
PHS2	1.0013	1.0003	1.0003
FEAT	1.0028	1.0002	1.0068
FEXP	1.0018	1.0019	1.0036
ADV	1.0006	1.0006	1.0019
σ_α	1.0329	1.0039	1.0321
σ_u	1.0191	1.0066	1.0246
α_1	1.0003	1.0003	1.0011
α_2	1.0031	1.0053	1.0026
α_3	1.0010	1.0014	1.0008
α_4	1.0017	1.0000	1.0007
α_5	1.0048	1.0005	1.0031

that if these computed values are below 1.2, then the sequence has converged. Since all of the values meet this criteria, the burn-in period of 500 is appropriate.

The second issue is addressed by plotting the autocorrelations and the partial autocorrelations for the successive draws of the parameter. Let z_t be the tth draw from the Gibbs sampler. Autocorrelation at the kth lag is the correlation between z_t and z_{t-k}. Partial autocorrelation at the kth lag is the correlation between z_t and z_{t-k} when their correlation with $z_{t-1}, \ldots, z_{t-k+1}$ is removed (Abraham and Ledolter 1983). It can be thought of as the partial regression coefficient ϕ_{kk} in

$$z_t = \phi_{k1} z_{t-1} + \ldots + \phi_{kk} z_{t-k} + u_t.$$

If the kth (and higher) autocorrelations and partial autocorrelations appear to be small, then a sample comprised of draws from every kth Gibbs loop will be an approximately uncorrelated sample which behave like independent and identically distributed draws from the posterior. Obviously, for a fixed number of Gibbs loops, the choice of k involves a tradeoff between sample size and degree of correlation among draws in the sample. Slow convergence (requiring a long burn-in period) also diminishes sample size for a given number of loops.

The plots for the autocorrelation function (shown in Figure 2) and the partial autocorrelation function (shown in Figure 3) use the same four parameters as in the convergence "tests". The dotted lines in these plots represent the 95% confidence region for the partial autocorrelation coefficients based on the assumption of no partial autocorrelation. If the autocorrelations and partial autocorrelations at lag k and above all appear to be insignificantly different from zero, then using every kth draw results in a

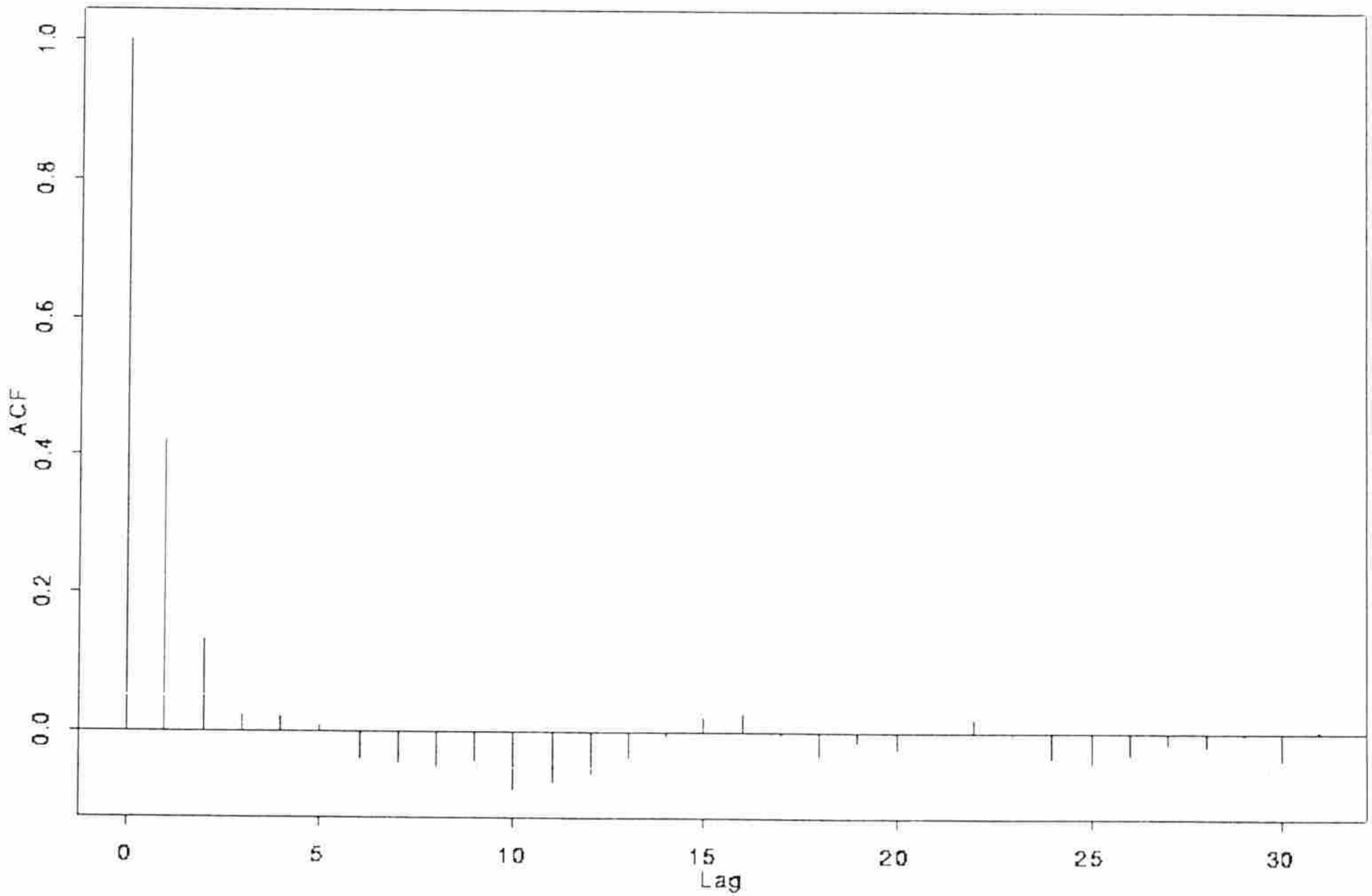

Figure 2. Autocorrelation plot of simulations draws of steak price effect on steak demand

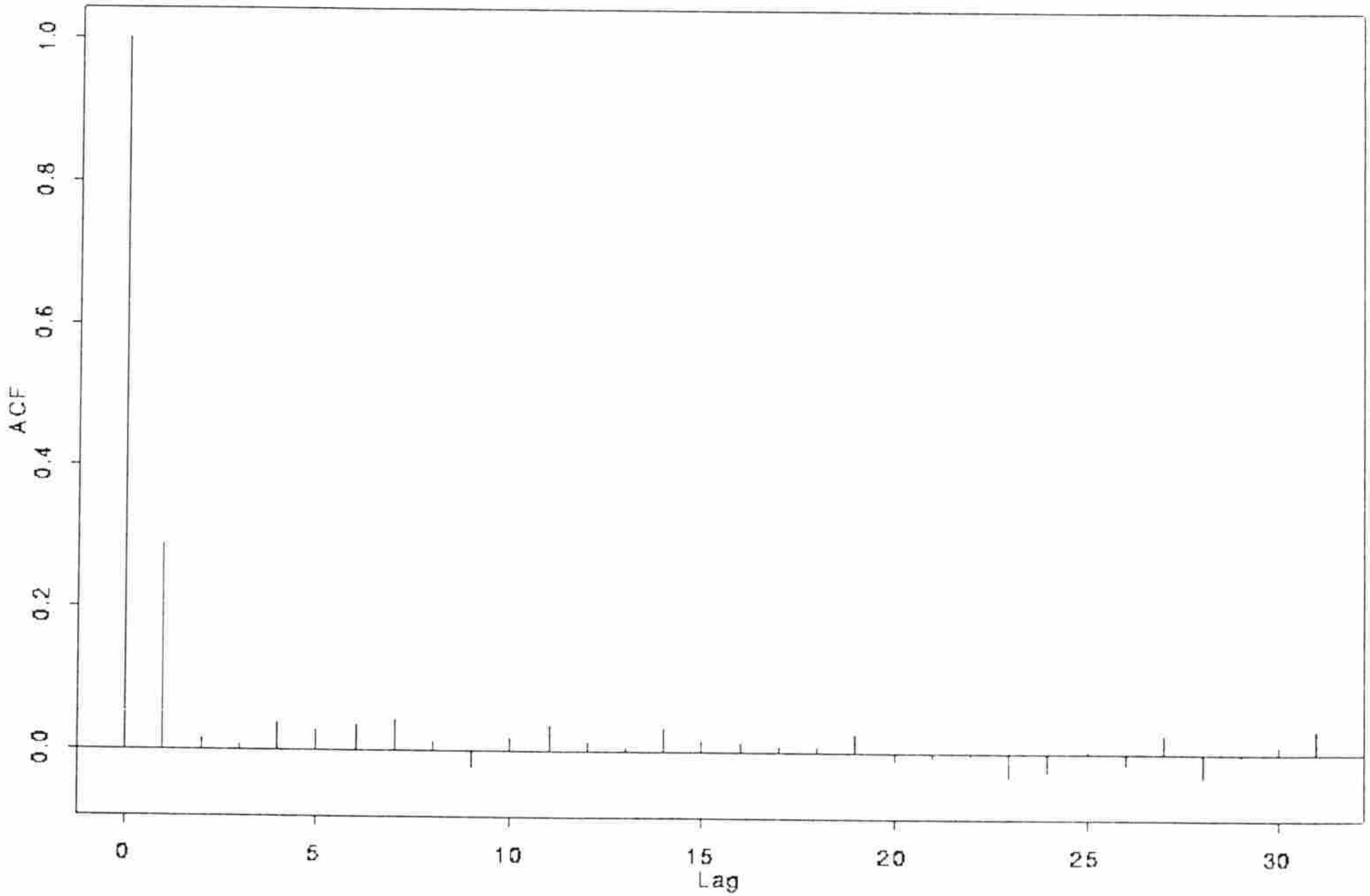

Figure 2. (continued) Autocorrelation plot of simulations draws of advertising effect on steak demand

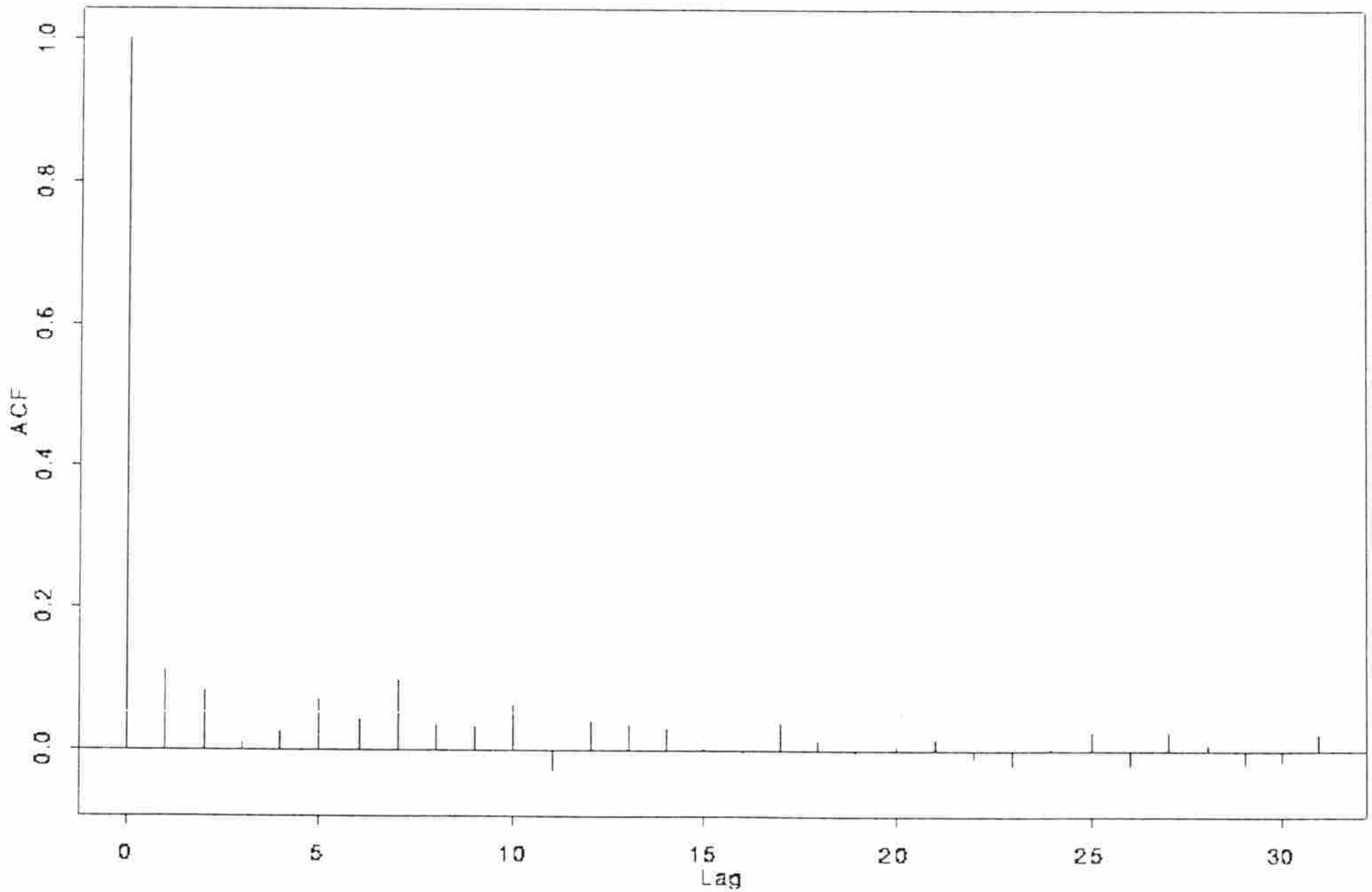

Figure 2. (continued) Autocorrelation plot of simulations draws of random effect for household one on steak demand

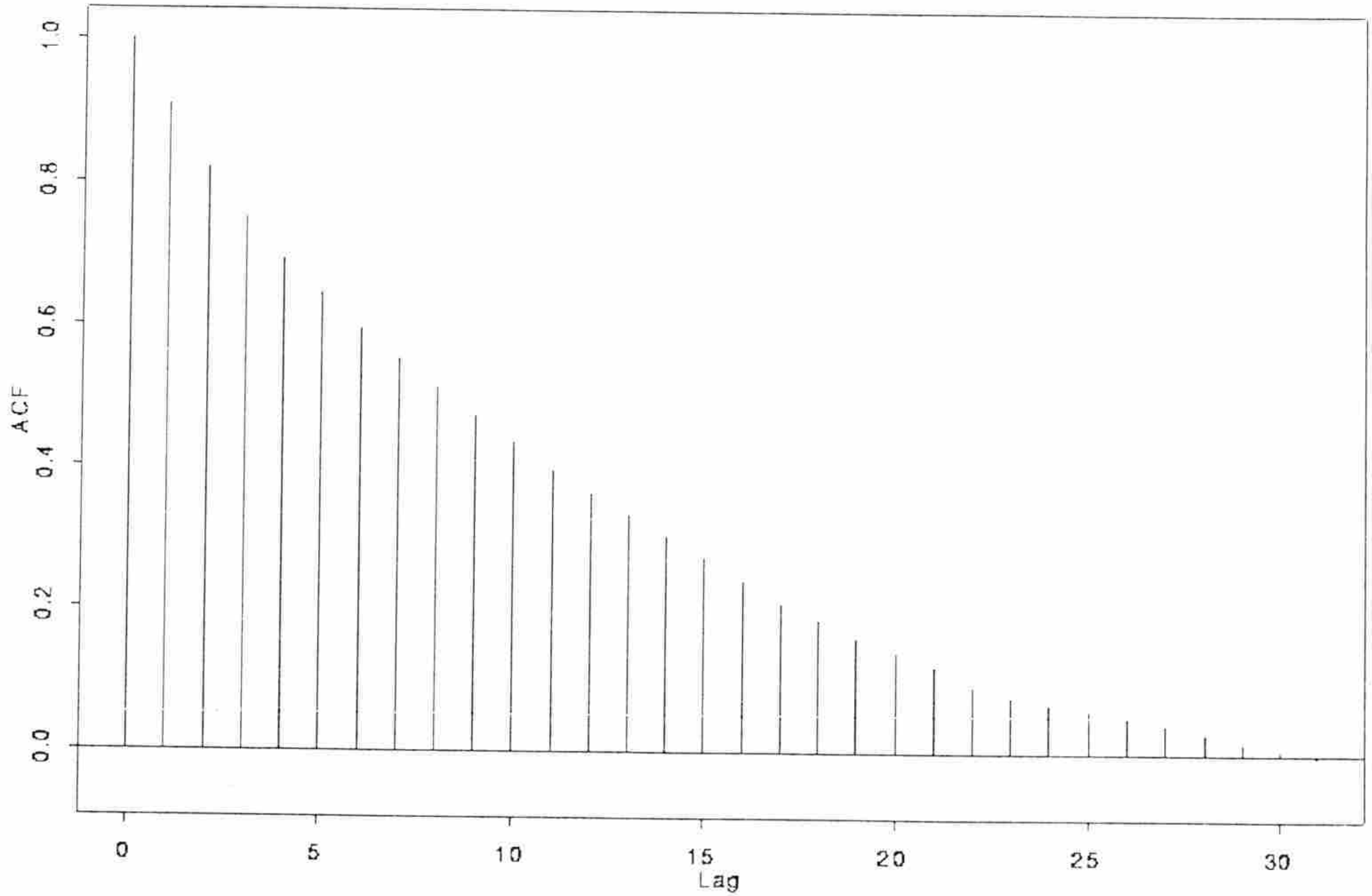

Figure 2. (continued) Autocorrelation plot of simulations draws of random effects variance on steak demand

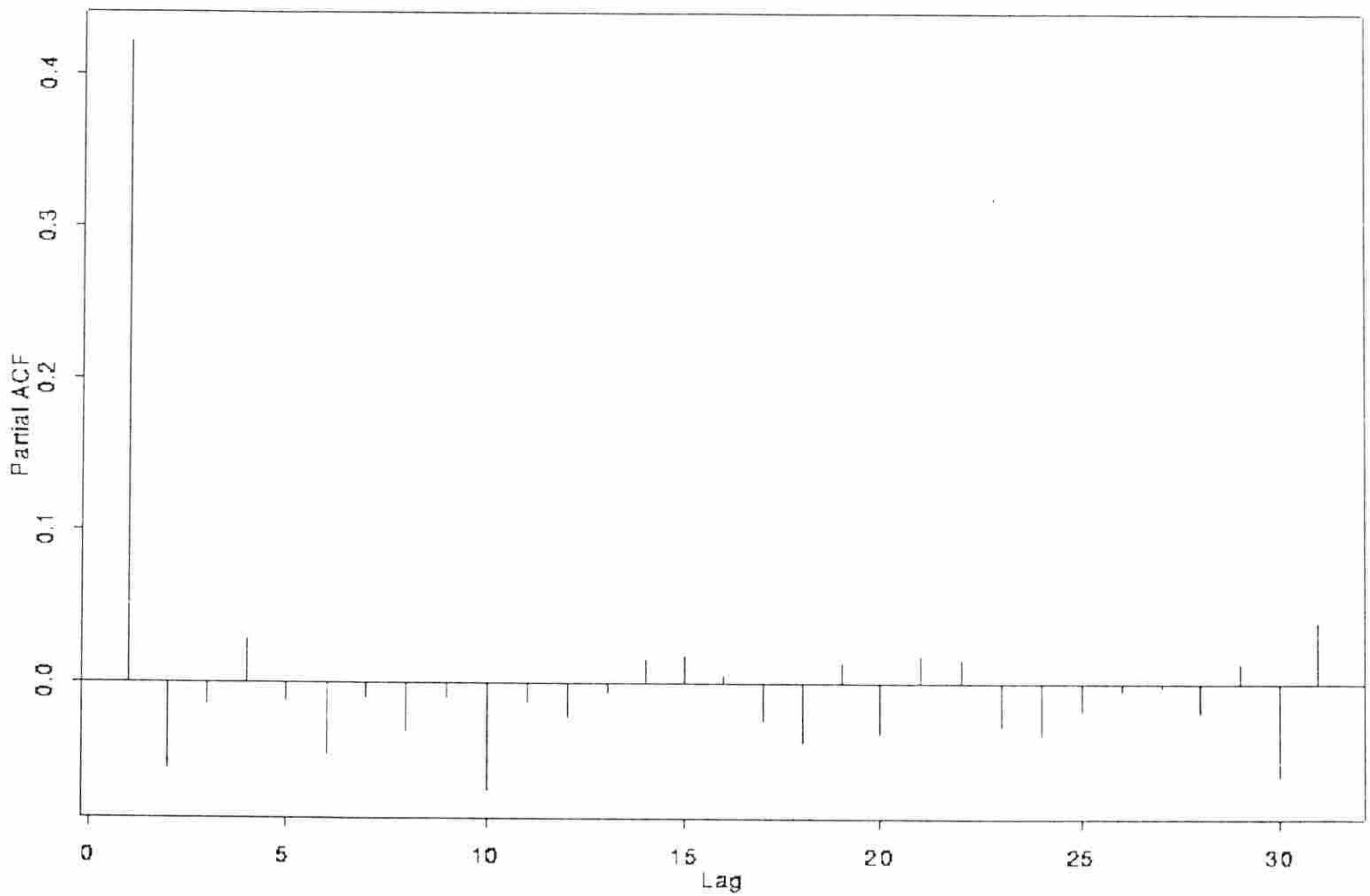

Figure 3. Partial autocorrelation plot of simulation draws of steak price effect on steak demand

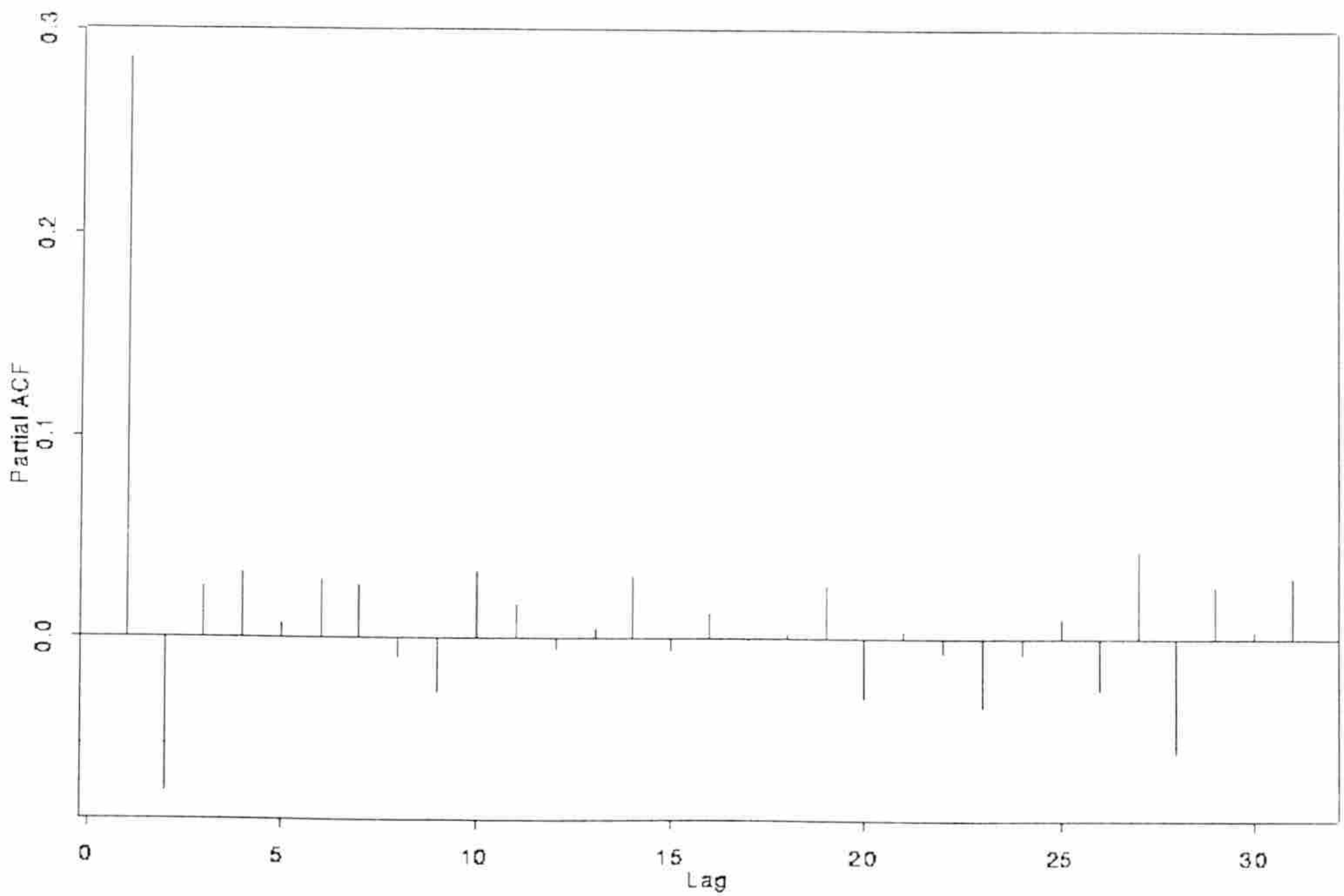

Figure 3. (continued) Partial autocorrelation plot of simulation draws of advertising effect on steak demand

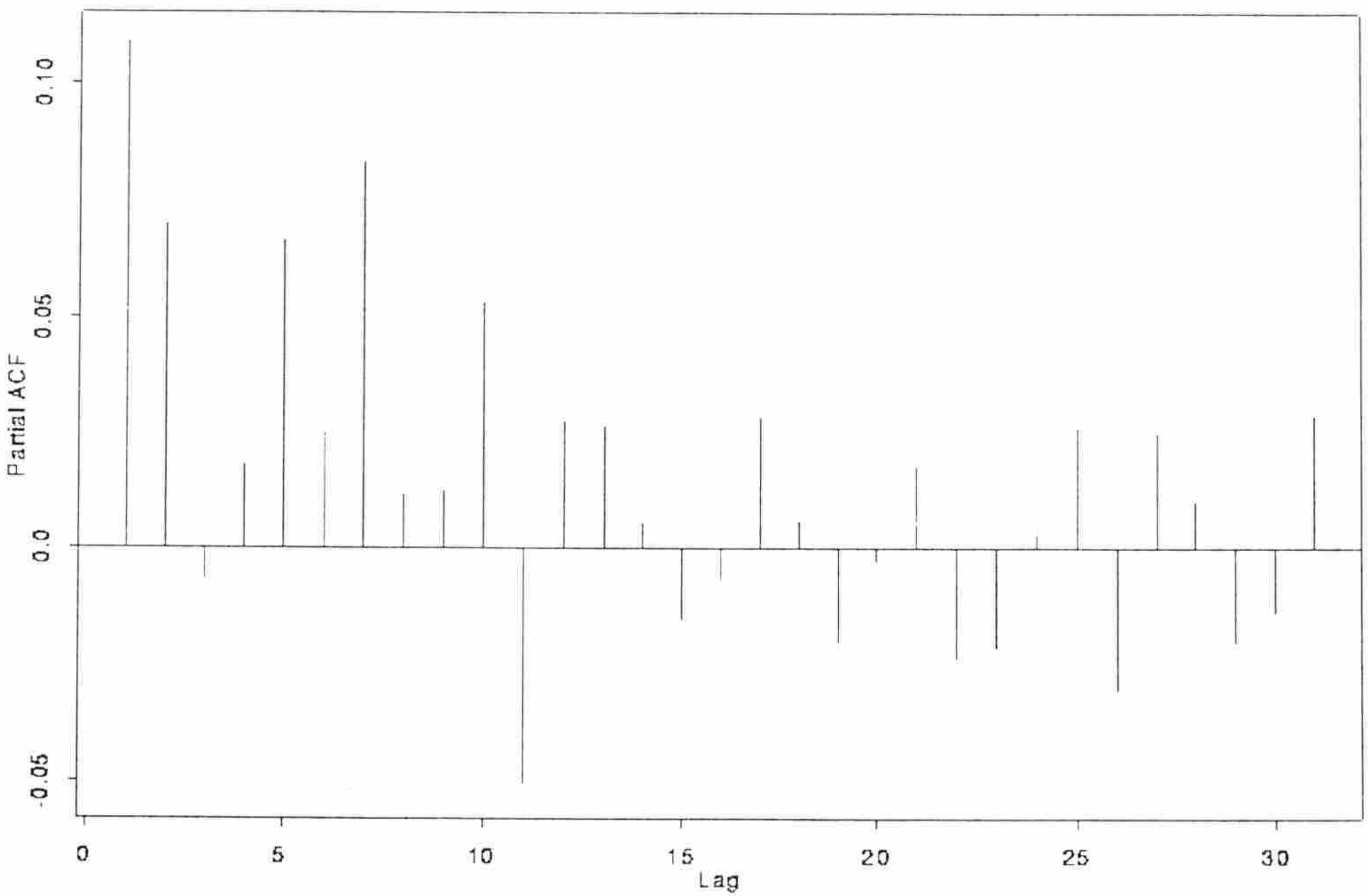

Figure 3. (continued) Partial autocorrelation plot of simulation draws of random effect for household one on steak demand

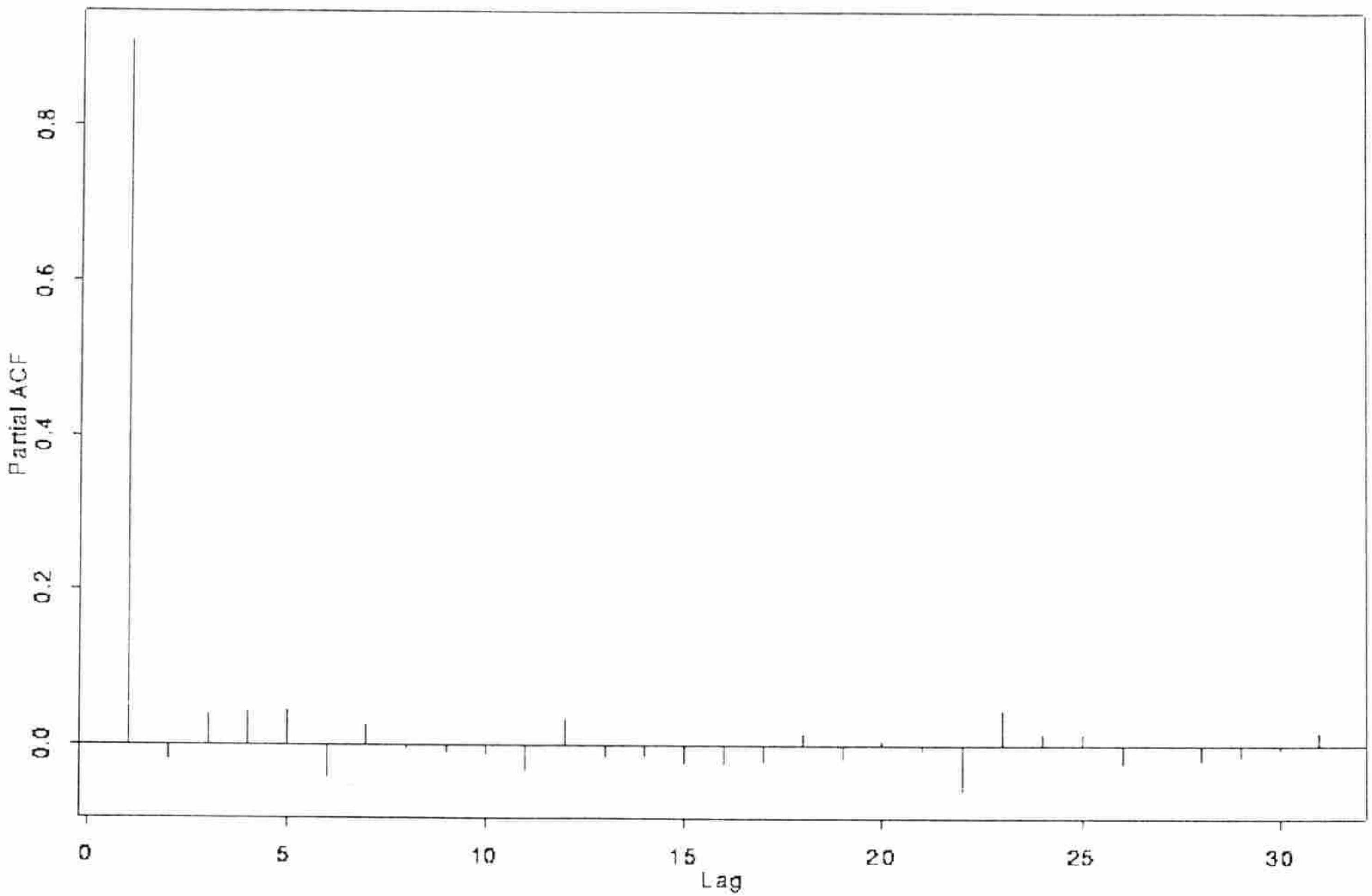

Figure 3. (continued) Partial autocorrelation plot of simulation draws of random effects variance on steak demand

sample in which the draws are essentially uncorrelated. For autocorrelation, the advertising effect shows no significant correlation after the first lag, while the plots of the other three do show significant correlation at the tenth lag or higher. The random effects variance shows significant correlation as high as the 25th lag. For the partial autocorrelations, all plots except for the random effect for household one appear to show no significant partial correlation among the draws after the second lag except for a few outliers. The plot for partial autocorrelation among the draws for the household effect shows significant partial correlation at lag nine. These plots are representative of those for the rest of the parameters. From analysis of these partial autocorrelation plots and others that are not presented here, every tenth draw was chosen in compiling the sample from the posterior distribution. Although the autocorrelation plots show that the correlation is still significant at the tenth lag for all but the advertising coefficient, a trade-off is made to obtain more information about the posterior distributions.[19]

EMPIRICAL RESULTS

The posterior means of the regressors, their respective probabilities of being positive, and their respective credible intervals[20] are presented in Tables 6, 7, and 8 for the steak model, the roast model, and the ground beef model, respectively.

All of the own-price effects are negative with estimated posterior probability one as is expected from demand theory. The cross-price effects are more interesting. For the steak model, decreases in prices of roast, ground beef, and chicken lead to an increase in demand for steak, while pork prices have the opposite effect. For the roast model, increases in steak and chicken prices increase roast beef demand, while increases in ground beef and pork prices reduce roast demand. Finally, in the ground beef model, increases in the prices of steak or pork increase ground beef demand while roast and chicken prices do the opposite. Each of the price effects is positive with high probability (greater than .90) for positive effects or positive with low probability (less than .10) for negative effects except for the pork price effect on roasts, so it is not evident whether this effect is positive or negative. Each of the food expenditure effects is positive with high probability indicating that an increase in food expenditures leads to an increase in demand for all three types of fresh beef.

The demand elasticities for these price effects and food expenditures along with their credible intervals and posterior probabilities of being elastic are presented in Table 9. In comparing the elasticities to other studies, Heien and Pompelli (1988) evaluated "partial" price and expenditure elasticities for the same three categories of fresh beef.[21]

Table 6. Estimates of Steak Demand Equation

Variable	Posterior Mean of Coefficient	Lower Limit for Credible Interval	Upper Limit for Credible Interval	Positive Posterior Probability[a]
MSE	-0.013222	-0.018074	-0.008396	0.0000
MSUE	-0.011085	-0.016468	-0.005878	0.0000
FSE	-0.007398	-0.013437	-0.001780	0.0147
FSUE	-0.004866	-0.011067	0.001078	0.0867
CH1	-0.070081	-0.098200	-0.042383	0.0000
CH2	-0.070019	-0.093740	-0.047660	0.0000
CH3	-0.044414	-0.067830	-0.020803	0.0013
SIZE	-0.009710	-0.030714	0.012905	0.2333
MHM	-3.988275	-176.3008	161.5809	0.4947
FHM	2.607498	-158.7200	164.5226	0.5147
MHA	-0.006140	-0.023000	0.009600	0.2333
FHA	-0.026056	-0.042133	-0.009697	0.0027
NW	-2.569157	-181.0936	165.3019	0.4880
HISP	0.555864	-159.1909	167.8181	0.4880
BAP	-6.129177	-179.1266	165.2602	0.4827
HAP	-4.926308	-172.0394	158.7678	0.4720
OWN	-0.678080	-171.1400	165.9305	0.5040
DISH	-4.121562	-168.4224	162.1078	0.4707
SPR	-0.004815	-0.005955	-0.004062	0.0000
RPR	-0.000966	-0.002097	0.000188	0.0787
GPR	-0.002183	-0.004244	-0.000082	0.0400
PPR	0.005902	0.005094	0.006696	1.0000
CPR	-0.004271	-0.006342	-0.002304	0.0000
FEXP	0.000097	0.000085	0.000109	1.0000
FEAT	0.000978	0.000956	0.000999	1.0000
PHS1	5.918556	-157.3091	169.2250	0.5093
PHS2	2.006923	-169.4332	166.3209	0.5067
ADV	0.000077	-0.000011	0.000169	0.9280

[a]This column is the P[coefficient > 0|Y], i.e. the posterior probability of being positive.

Table 7. Estimates of Roast Demand Equation

Variable	Posterior Mean of Coefficient	Lower Limit for Credible Interval	Upper Limit for Credible Interval	Positive Posterior Probability[a]
MSE	-0.005600	-0.013371	0.000016	0.0480
MSUE	-0.006970	-0.012838	-0.001395	0.0213
FSE	0.004099	-0.002275	0.010640	0.8427
FSUE	0.009374	0.002902	0.016353	0.9893
CH1	-0.048434	-0.080406	-0.014516	0.0053
CH2	-0.009555	-0.037867	0.016548	0.2640
CH3	0.015131	-0.012657	0.042087	0.8240
SIZE	-0.079579	-0.104694	-0.005289	0.0000
MHM	1.226968	-167.2566	176.0505	0.5107
FHM	7.696587	-166.4600	176.8281	0.5253
MHA	0.015343	-0.003434	0.033782	0.9053
FHA	0.118735	0.099236	0.138607	1.0000
NW	0.359382	-166.5579	164.2313	0.5147
HISP	3.748974	-172.8121	168.7577	0.5240
BAP	2.038228	-172.0197	163.1740	0.5293
HAP	3.109262	-170.9639	169.9879	0.5053
OWN	3.502579	-158.3709	166.1212	0.5413
DISH	-1.058219	-164.0937	168.7748	0.4840
SPR	0.005337	0.004406	0.006215	1.0000
RPR	-0.010258	-0.011585	-0.008957	0.0000
GPR	-0.008670	-0.011918	-0.005947	0.0000
PPR	-0.000222	-0.001148	0.000723	0.3373
CPR	0.002000	-0.005739	0.004505	0.9173
FEXP	0.000053	0.000040	0.000065	1.0000
FEAT	0.000890	0.000865	0.000913	1.0000
PHS1	2.640587	-167.3125	171.4975	0.5213
PHS2	0.083074	-163.2240	166.5323	0.4960
ADV	0.000048	-0.000052	0.000149	0.7827

[a]This column is the P[coefficient > 0|Y], i.e. the posterior probability of being positive.

Table 8. Estimates of Ground Beef Demand Equation

Variable	Posterior Mean of Coefficient	Lower Limit for Credible Interval	Upper Limit for Credible Interval	Positive Posterior Probability[a]
MSE	-0.090500	-0.101772	-0.079124	0.0000
MSUE	-0.082149	-0.094294	-0.070130	0.0000
FSE	-0.051594	-0.067126	-0.363777	0.0000
FSUE	-0.060289	-0.076280	-0.045518	0.0000
CH1	0.065830	-0.002044	0.130445	0.9520
CH2	0.059629	0.005050	0.117619	0.9707
CH3	-0.156218	-0.208705	-0.094127	0.0000
SIZE	0.336730	0.282264	0.386292	1.0000
MHM	-0.342108	-157.9782	167.7033	0.5027
FHM	-0.289887	-172.3183	176.0245	0.4920
MHA	0.020134	-0.017392	0.057642	0.8173
FHA	-0.114175	-0.153281	-0.076039	0.0000
NW	0.843516	-178.4790	162.1213	0.5200
HISP	7.991289	-148.7353	169.3983	0.5320
BAP	-0.822301	-171.6273	164.9984	0.4920
HAP	3.205693	-159.5056	161.0500	0.5200
OWN	7.794849	-161.3718	177.9330	0.5373
DISH	0.499620	-165.1170	168.2714	0.4853
SPR	0.003053	0.001444	0.004709	0.9987
RPR	-0.002918	-0.005113	-0.000623	0.0120
GPR	-0.017766	-0.022769	-0.013226	0.0000
PPR	0.008430	0.006615	0.010242	1.0000
CPR	-0.005444	-0.010205	0.001001	0.0227
FEXP	0.000321	0.000307	0.000335	1.0000
FEAT	0.001792	0.001744	0.001837	1.0000
PHS1	7.747724	-166.2317	173.5339	0.5147
PHS2	-4.543151	-169.1314	161.4845	0.4773
ADV	0.000750	0.000541	0.000963	1.0000

[a]This column is the P[coefficient > 0|Y], i.e. the posterior probability of being positive.

Table 9. Estimates of expenditure and price elasticities at sample and posterior means[a]

Variable	Elasticity Estimate	Lower Limit for Credible Interval	Upper Limit for Credible Interval	Elasticity Probability[b]
Steak Demand Model:				
SPR	-2.612	-3.036	-2.204	1.0000
RPR	-0.352	-0.763	0.068	0.0013
GPR	-0.573	-1.115	-0.021	0.0947
PPR	3.395	2.931	3.853	1.0000
CPR	-0.754	-1.120	-0.407	0.1280
FEXP	1.468	1.283	1.649	1.0000
Roast Demand Model:				
SPR	3.462	2.858	4.031	1.0000
RPR	-4.466	-5.043	-3.899	1.0000
GPR	-2.723	-3.515	-1.868	1.0000
PPR	-0.153	-0.790	0.497	0.0133
CPR	0.422	-0.121	0.952	0.0400
FEXP	0.950	0.727	1.185	0.3573
Ground Beef Demand Model:				
SPR	0.732	0.346	1.129	0.1160
RPR	-0.469	-0.823	-0.100	0.0093
GPR	-2.062	-2.642	-1.535	0.9987
PPR	2.143	1.682	2.604	1.0000
CPR	-0.425	-0.797	-0.079	0.0013
FEXP	2.148	2.055	2.242	1.0000

[a]For example, the steak price elasticity with respect to the demand for steak is

$$\xi = \bar{\beta}_{SPR}\frac{\overline{SPR}}{\overline{SQ}\,\overline{PC}}.$$

[b]This column is P[|estimate of elasticity| > 1], i.e. the posterior probability that demand is elastic.

Their elasticity estimates were computed using an almost ideal demand system, so the implications may not be the same, but they do offer valuable insights.

Each of the own-price effects are elastic (greater than one in absolute value) which means that a small change in price will produce a big change in demand. These elasticities are all greater in absolute value than the aggregate elasticity estimate in Jensen and Schroeter, -1.250. This is to be expected because a good that is more narrowly defined should have more elastic demand. For the expenditure elasticities, the credible interval for steak demand in Table 9, (1.283, 1.649), does not cover Heien and Pompelli's estimate of 1.14, although the expenditure elasticity on steak demand for both studies is elastic. For roast demand, the credible interval, (0.727, 1.185) also does not cover their estimate, 1.37. It is not apparent whether expenditure elasticity for roasts is elastic or inelastic. For ground beef demand, the credible interval, (2.055, 2.242), shows that the expenditure elasticity on ground beef demand is elastic, but their estimate, .69, is inelastic.[22]

They also found many cross-price effects to be negative, although not the same ones as in this analysis. All of the negative cross-price effects in this study appear to be inelastic except for the effect of ground beef prices on roast demand. Of the cross-price effects that are positive, only two are inelastic. They are the effect of chicken prices on roast beef demand and the effect of steak prices on ground beef demand. It also appears that steak and pork chops are strong substitutes with a high positive elasticity.

Of the household demographic variables, education, employment status, number of children, household size, and age of head of household appear to have a significant effect on demand for one or more of the types of fresh beef per standard person.[23] The other demographic variables, which include homemaker variables, race variables, the home ownership variable, and the dishwasher variable, do not appear to have a significant effect on demand for any type of fresh beef.

For the steak model, an increase in wage rates (proxied by education attainment level) causes a decrease in steak demand. The wage rate effect is stronger for employed heads of households, which is supported by household production theory, and is also stronger for male heads of households than for female heads which is also expected because historically male wages have been higher than those of females with the same educational level. Also increases in the number of children decreases demand per standard person, which means that steak preparation and child care activities are competing activities. As expected, this effect is stronger if the household has more younger children. The number of standard persons in a household does not appear to have an effect on standardized steak demand, which means that the economies of scale do not appear to be in force with steak preparation. The age of the male head of household appears to have an insignificant effect on steak demand, while a higher age for the female head of household reduces the demand for steak.

For the roast model, an increase in the wage rate for a male head of household causes a decrease in roast demand, while an increase in the wage rate for a female head of household increases roast demand. Surprisingly, the negative effect is stronger for unemployed male heads of household than for those that are employed. As expected, the wage rate for male heads of household have a strong negative effect over that of female heads. The number of children in the youngest age group (zero to six years) has a definite negative effect on demand for roast beef per standard person. As the age of the children increase, the posterior mean increases with a positive posterior mean for the effect of the number of children in the twelve to eighteen year old group supporting the hypothesis, although the number of children in the two oldest age groups do not show a significant effect. An increase in the household's number of standard persons results in a decrease in demand for roasts per standard person. This is a surprising result because of the expectation that preparing roasts for more people would only require a small amount of extra preparation time. The age of the heads of household for both males and females has a positive effect on demand for roast beef, with female heads having a much larger increase in roast beef demand than male heads as their age increases.

For the ground beef model, an increase in wage rates have a very strong negative effect on demand for ground beef. Once again, the wage rate for male heads of household has a stronger negative effect than those for female heads. The wage rate effect for employed male heads of household is not as strong as that for unemployed males heads, while it is opposite for the female heads. The two younger age categories show an increase in demand per standard person as the number of children in those categories increases, while the number of children in the oldest age category results in the opposite. This is the opposite effect than what is expected by the hypothesis. The meaning of these results is that for younger children, child care and ground beef preparation are complementary activities, but as the children become older, the activities start to compete with each other more. Unlike the roast beef demand, an increase in the number of standard persons results in an increase in demand for ground beef per standard person. It appears that the economies of scale are in effect with the ground beef demand. This is expected because the increased cost of preparing more ground beef doesn't increase as fast as the size of the products being prepared. An example of this could be that the meal preparer cooks ground beef for a casserole. If he or she were to prepare more, the only extra time involved would be including bigger portions of each ingredient, which does not require much extra effort and time. An increase in the age of female heads of household reduces the ground beef demand while the opposite result was found for male heads; although, it was not as significant.

The coefficients for the random effects are interpreted as the effect of each individual household after the influences of all of the other explanatory variables have been accounted for. Most of the households did not have any significant effect on demand for fresh beef. For the roast model, only one household has a posterior probability of being positive that is one; therefore, this household is the only one that can be said for sure to have a positive impact on demand for roasts. All other households in either of the other two models have posterior probabilities of being positive that are less than one. Also, no household in any of the three models has a posterior probability of being negative equal to one; therefore, no household had a definite negative impact on demand for any type of fresh beef. One problem with making inferences for a random effect model with panel data is that most of the α_i's are accounted for by the household specific explanatory variables which do not vary over time.

Four dummy variables were included to test whether there are any systematic differences between ad panels or between phases of the experiment that are unaccounted for by other variables in the model. For all three models, the posterior probabilities of being positive were all close to .5 leading to the conclusion that panel membership or the phase of the test did not, by themselves, affect the demand for fresh beef. Implications of the FEAT variable are discussed in the possible extensions chapter.

The main variable to test the effect of television advertising is ADV, which was defined earlier as a 12-month, second-order polynomial distributed lag in advertising GRP's. Each of the advertising effects show a positive effect on demand for that specific type of fresh beef. The posterior probability of being positive is highest for ground beef demand with a probability of 1.0, while the steak demand advertising coefficient also appears to have a significant probability of being positive, .928. The positive effect of advertising on roast demand does not appear to be as significant with the probability of being positive only .783. Histograms plotting draws for the advertising effect for each type of fresh beef demand are show in Figure 4.

While it does appear that the advertising did have a positive effect, the question of the magnitude of the effect remains. Tables 10 and 11 show the total predicted change of seasonally adjusted demand per standard person with advertising present versus no advertising. This is computed for each category of fresh beef, panel, and four-week time period. The predicted increase in demand is measured in pounds per four-week period

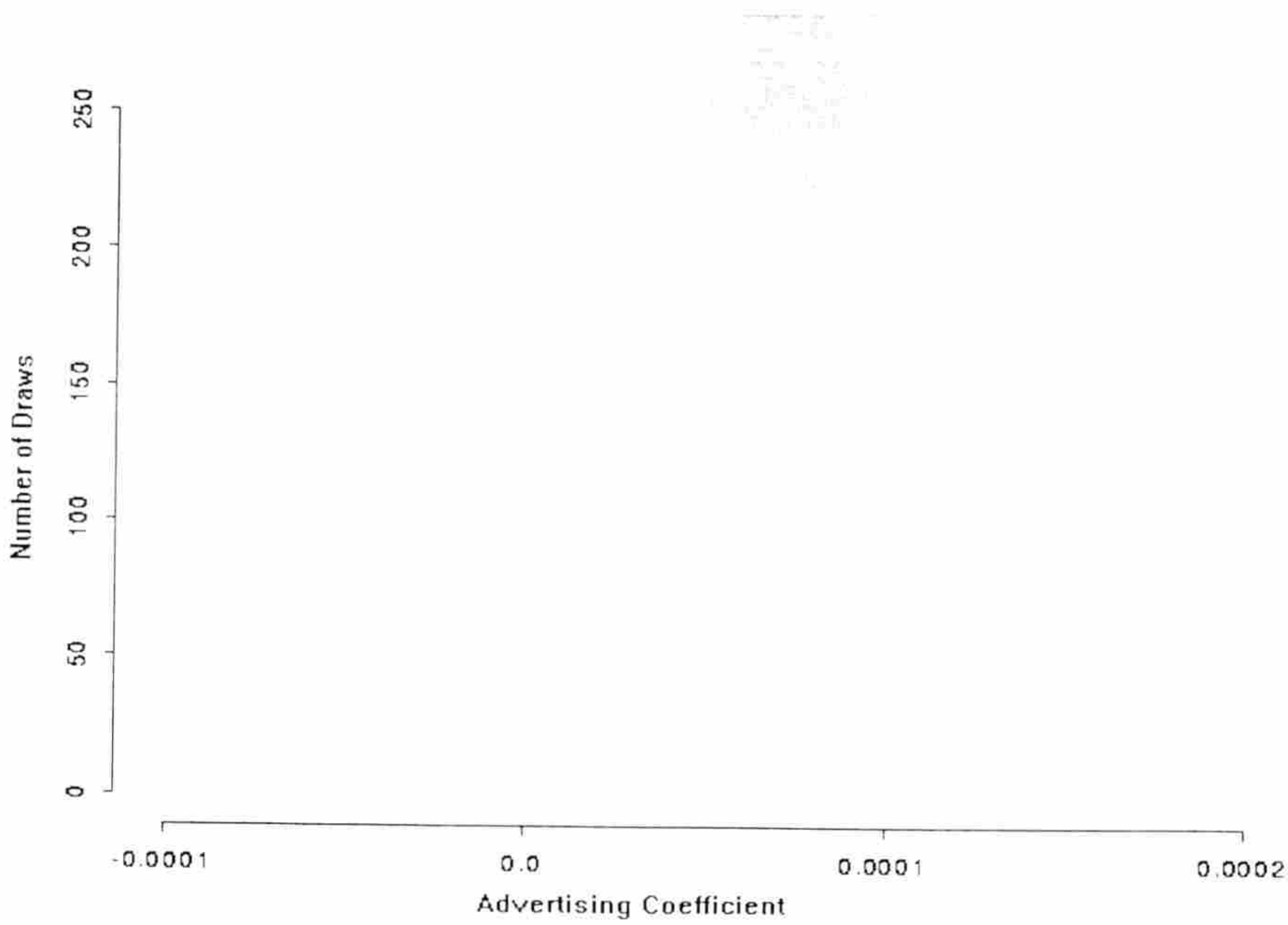

Figure 4. Histogram of simulation draws for advertising effect on steak demand

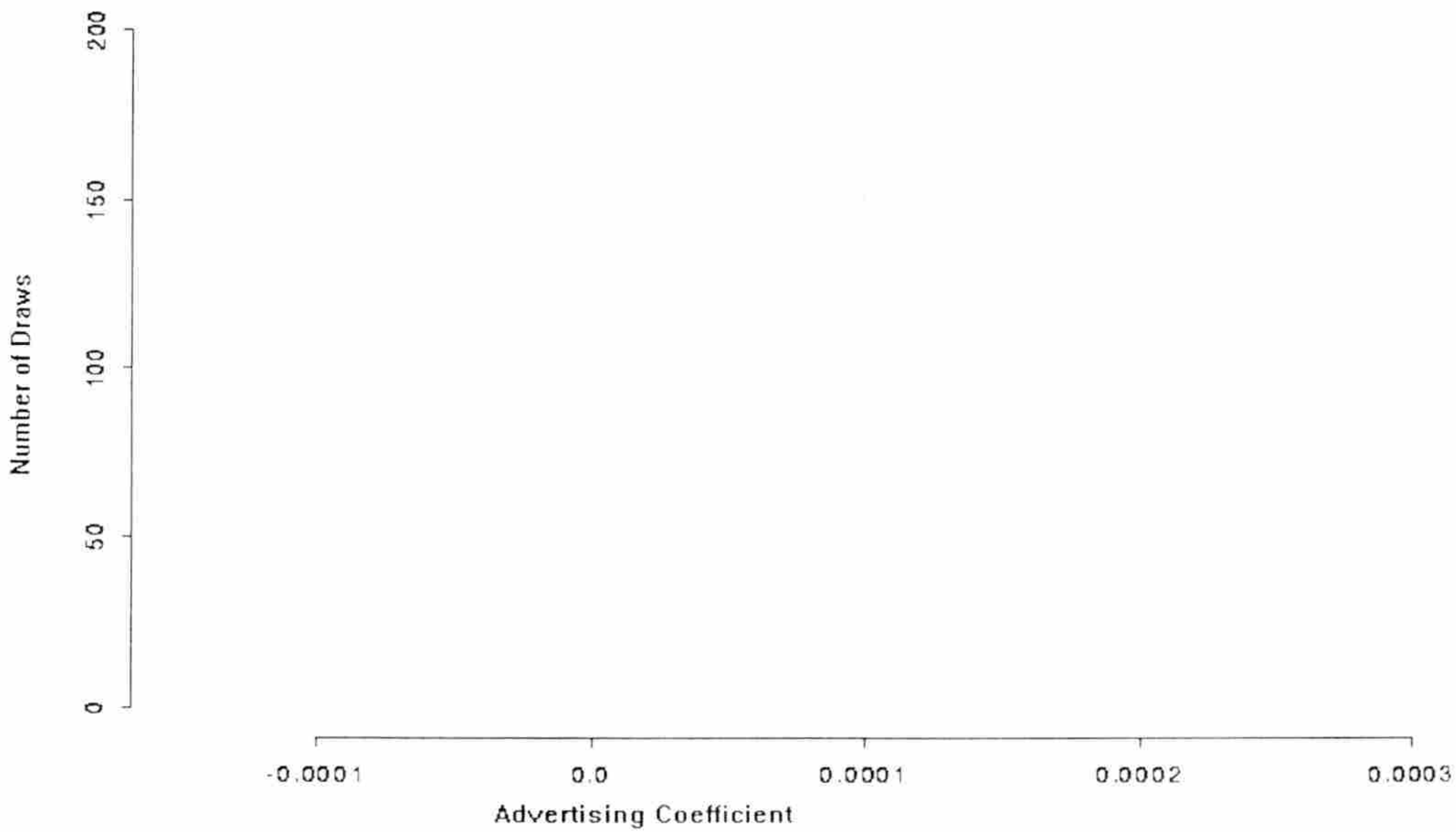

Figure 4. (continued) Histogram of simulation draws for advertising effect on roast demand

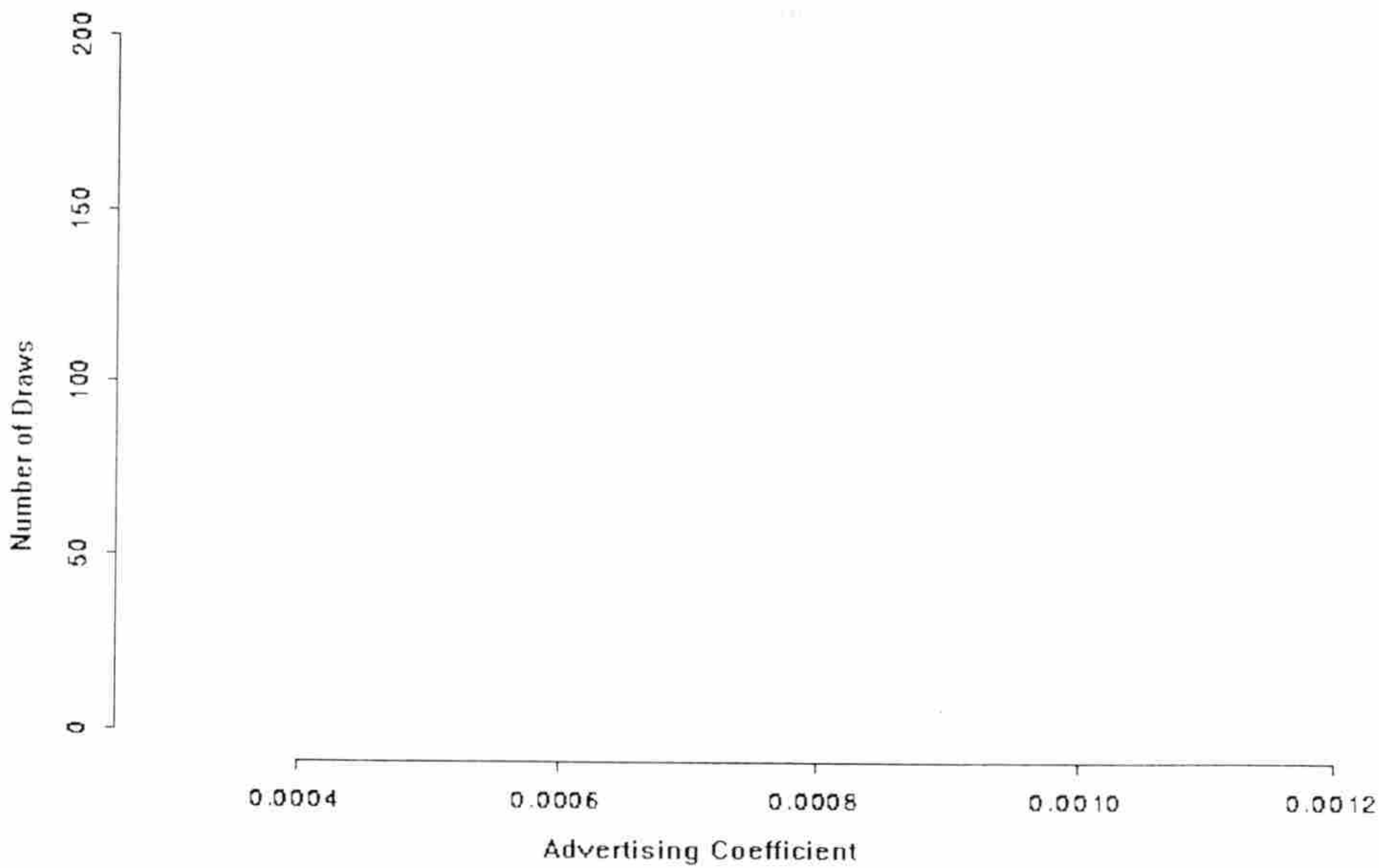

Figure 4. (continued) Histogram of simulation draws for advertising effect on ground beef demand

Table 10. Effect of advertising versus no advertising for base-ad panel[a]

Time Period	Steak Demand Model	Roast Demand Model	Ground Beef Demand Model
1	0.0000	0.0000	0.0000
2	0.0000	0.0000	0.0000
3	0.0000	0.0000	0.0000
4	0.0000	0.0000	0.0000
5	0.0009	0.0006	0.0089
6	0.0021	0.0013	0.0208
7	0.0031	0.0019	0.0304
8	0.0043	0.0027	0.0421
9	0.0056	0.0035	0.0550
10	0.0066	0.0041	0.0642
11	0.0072	0.0047	0.0699
12	0.0074	0.0046	0.0719
13	0.0081	0.0050	0.0787
14	0.0082	0.0051	0.0805
15	0.0079	0.0049	0.0772
16	0.0071	0.0044	0.0689
17	0.0061	0.0038	0.0596
18	0.0064	0.0040	0.0628
19	0.0071	0.0044	0.0697
20	0.0087	0.0054	0.0848
21	0.0101	0.0063	0.0987
22	0.0118	0.0074	0.1155
23	0.0128	0.0080	0.1248

[a]Values are computed at the posterior mean of β_{ADV} and multiplied by ADV_{it}.

Table 11. Effect of advertising versus no advertising for heavy-ad panel[a]

Time Period	Steak Demand Model	Roast Demand Model	Ground Beef Demand Model
1	0.0000	0.0000	0.0000
2	0.0000	0.0000	0.0000
3	0.0000	0.0000	0.0000
4	0.0000	0.0000	0.0000
5	0.0018	0.0011	0.0178
6	0.0043	0.0027	0.0415
7	0.0080	0.0050	0.0777
8	0.0118	0.0073	0.1149
9	0.0156	0.0097	0.1520
10	0.0201	0.0125	0.1958
11	0.0241	0.0150	0.2351
12	0.0275	0.0171	0.2686
13	0.0311	0.0194	0.3032
14	0.0327	0.0204	0.3194
15	0.0325	0.0202	0.3171
16	0.0304	0.0189	0.2963
17	0.0268	0.0167	0.2610
18	0.0241	0.0150	0.2353
19	0.0212	0.0132	0.2064
20	0.0199	0.0124	0.1941
21	0.0182	0.0113	0.1772
22	0.0164	0.0102	0.1604
23	0.0152	0.0095	0.1487

[a]Values are computed at the posterior mean of β_{ADV} and multiplied by ADV_{it}.

per standard person. All of the predicted values are higher for the heavy ad panel than for the base ad panel, so the heavy ad campaign will be examined.

For each demand model, the highest predicted increase in demand occurs in the 14th time period. They are .0327 pounds for the steak, .0204 pounds for the roasts, and .319 pounds for ground beef. The effectiveness of the advertising campaign on both steak and roast beef appears to be very small, but it does appear that advertising had a significant economic impact on the demand for ground beef. A plot of exposure level with the effect of advertising versus no advertising for the heavy-ad panel is shown in Figure 5.

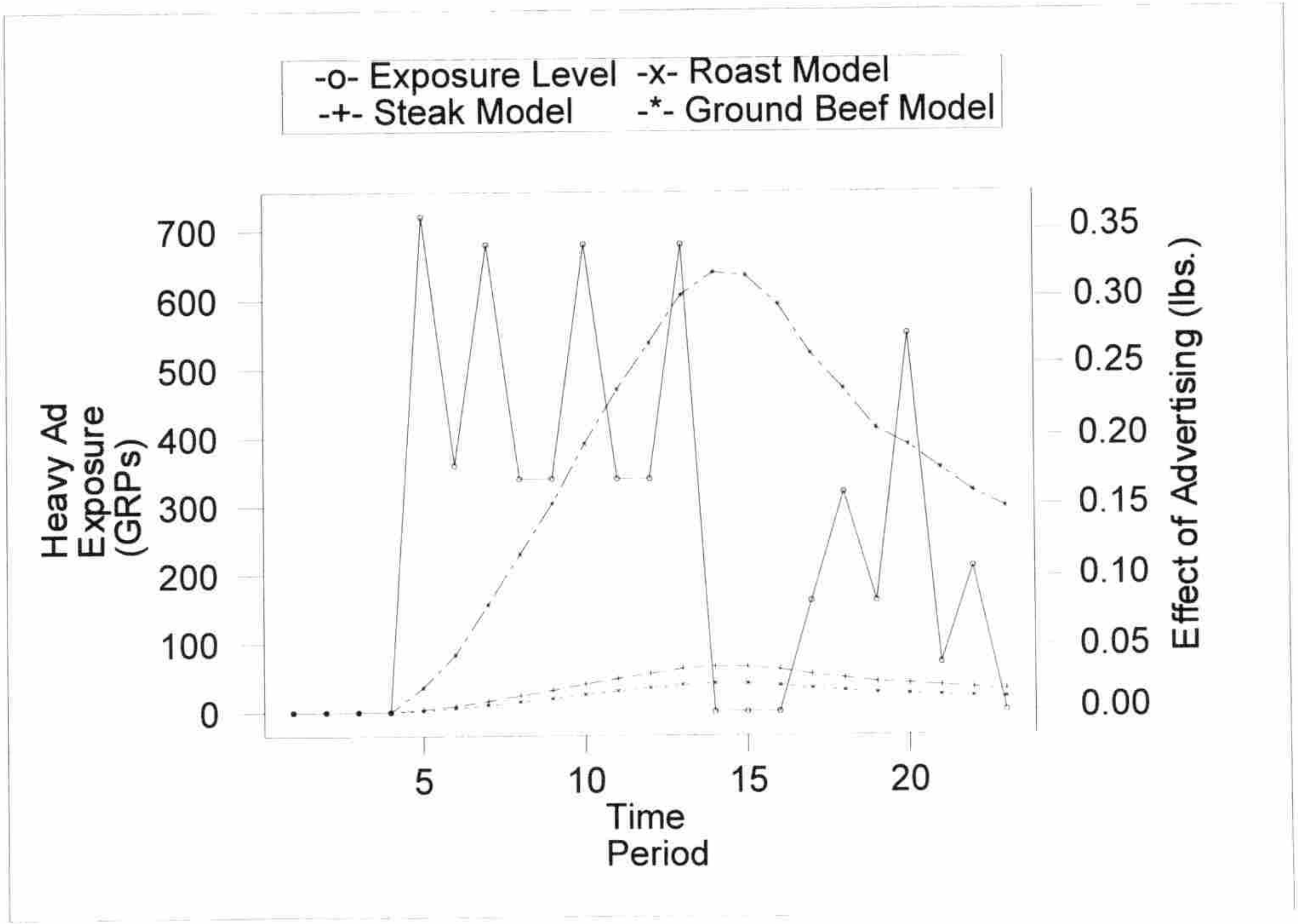

Figure 5. Plot of advertising exposure for heavy-ad panel vs. effect of advertising

DRAWBACKS OF THE DATA SET

Although the Grand Junction experiment was very extensive and produced a large, household-level data set, there were still several shortcomings of the data and the experimental design.

The biggest drawback is the uncertainty about the kinds and amounts of advertising to which panel households were actually exposed. Regarding the test advertising itself, there is no information on whether the base-ad and heavy-ad panel households actually viewed the television advertisements available to them via cable and no guarantee that control panel households did not see the ads in the homes of friends or neighbors who belonged to the base-ad or heavy-ad panel. Regarding other types of advertising, there is no information on advertising received through other national media (e.g. radio or magazines), only very limited information on in-store or local media advertising, and no information on "negative" advertising. Negative advertising, in this context, would include public service announcements or personal medical advice counseling against excessive red meat consumption.

Another major drawback is that income is reported in categorical form only and there are not separate measures of wage and non-wage income. For this reason, educational attainment level is used as a proxy for the wage and food store expenditures are used as an income proxy.

The advertisements are intended to increase the demand for beef products. While there is extensive information on purchases of unprepared fresh beef products, households probably consumed other types of beef products, too. There is no data available on consumption of prepared beef products or consumption of beef away from home. Therefore, these models do not account for total consumption of beef.

Other potentially interesting aspects of beef demand that could not be tested are regional effects and urban-rural effects because the data is from a single city in the western region of the United States. Another drawback is that, due to the racial/ethnic composition of Grand Junction's population, most of the households in the experiment's panels were of the Caucasian race and were non-Hispanic. Because of this, it is difficult to test for racial and ethnic effects.

The last problem is that the demographic data are all reported in categorical form. This led to many compromises in the definition of variables. For example, a head of household's years of schooling had to be guessed on the basis of categorizations like: "graduated from high school", "completed some years of college", "graduated from college", etc. Similarly, ages of household members were known only to be within certain ranges. The sexes of children and of certain adult household members were not reported in the data. While the total number of children in each household was known, in some instances, the number of children falling within each of three age ranges could only be guessed. Obviously, this kind of necessary guesswork introduces error into the measurement of the explanatory variables.

POSSIBLE EXTENSIONS

This section briefly discusses possible extensions or revisions to the model. Several different ideas will be addressed including seemingly unrelated regression, an autoregressive model, variable transformation, non-normal error structure, testing structural changes over time, revision of the "featured" items variable, and different approaches to incorporating advertising.

Seemingly Unrelated Regression

The seemingly unrelated regression (SUR) model is a multi-equation model with no simultaneity (that is, no endogenous explanatory variables), but error terms are correlated across equations (Percy 1992). A SUR version of the beef demand models can be written as follows:

$$y = \alpha + X^{*}\beta + u;\ \alpha \sim \text{MVN}(0, \Gamma_{\alpha}),\ u \sim \text{MVN}(0, \Gamma_{u})$$

where

$$y = \begin{bmatrix} SQPC \\ RQPC \\ GQPC \end{bmatrix},\ X^{*} = \begin{bmatrix} X & 0 & 0 \\ 0 & X & 0 \\ 0 & 0 & X \end{bmatrix},\ = \begin{bmatrix} \alpha_{steak} \\ \alpha_{roast} \\ \alpha_{ground} \end{bmatrix},\ u = \begin{bmatrix} u_{steak} \\ u_{roast} \\ u_{ground} \end{bmatrix},\ \text{and } \beta = \begin{bmatrix} \beta_{steak} \\ \beta_{roast} \\ \beta_{ground} \end{bmatrix}.$$

The analysis reported in this thesis implicitly assumes that Γ_u takes the form:

$$\Gamma_u = \begin{bmatrix} \sigma^2_{u,steak} I & 0 & 0 \\ 0 & \sigma^2_{u,roast} I & 0 \\ 0 & 0 & \sigma^2_{u,ground} I \end{bmatrix}.$$

If, on the other hand, error terms from the steak, roast, and ground beef models are correlated, certain off diagonal elements of Γ_u will be nonzero. The change in the distribution results in different conditional distributions for $\Gamma | \alpha, \beta, z, Y$, and $\beta | \alpha, \Gamma, z, Y$, where $\Gamma = \Gamma_u + \Gamma_\alpha$ and

$$\Gamma_\alpha = \begin{bmatrix} \sigma^2_{\alpha,steak} I & 0 & 0 \\ 0 & \sigma^2_{\alpha,roast} I & 0 \\ 0 & 0 & \sigma^2_{\alpha,ground} I \end{bmatrix}.$$

Percy suggests parameterizing the model in terms of the precision matrix Ψ,[24] rather than the variance-covariance matrix Γ, and using Jeffrey's invariant prior:

$$f(\beta, \Psi) \propto |\Psi|^{-(3k+1)/2}.$$

The conditional priors can be derived from this information in a manner similar to the method used in the original model.

The motivation for using SUR is that many times consumers allocate income to beef in general and then use that beef budget to allocate purchases for the three categories of beef.

Autoregressive Model

For an autoregressive model the following model is used:

$$Y_{it} = \alpha_i + \beta' x_{it} + u_{it} \text{ where } u_{it} - \phi_1 u_{i,t-1} - \dots - \phi_n u_{i,t-n} = a_{it};\ a_{it} \sim N(0, \sigma_a^2) \qquad (5)$$

with $i = 1, \dots, I$ and $t = n + 1, \dots, T$. Let

$$u = (u_{1,n+1}, u_{1,n+2}, \dots, u_{1,T}, \dots, u_{I,n+1}, \dots, u_{IT})',$$

$$U=\begin{bmatrix} u_{1,n} & \cdots & u_{1,1} \\ u_{1,n+1} & \cdots & u_{1,2} \\ \vdots & & \vdots \\ u_{1,T-1} & \cdots & u_{1,T-n} \\ \vdots & & \vdots \\ u_{I,n} & \cdots & u_{I,1} \\ \vdots & & \vdots \\ u_{I,T-1} & \cdots & u_{I,T-n} \end{bmatrix}_{I(T-n)\times n},$$

$\phi = (\phi_1, ..., \phi_n)'$, and

$a = (a_{1,n+1}, a_{1,n+2}, ..., a_{1,T}, ..., a_{I,n+1}, ..., a_{IT})'$.

Rewriting (5) in matrix form results in

$$u - U\phi = a \text{ or } u = U\phi + a.$$

Using results from Chib (1993), the resulting conditional distribution is:

$$\phi|\alpha, \beta, \sigma, z, Y \sim \text{MVN}(\phi^*, (\Phi^*)^{-1})$$

where

$$\phi^* = (\Phi^*)^{-1} (\Phi_0\phi_0 + \sigma_u^{-2}U'u)$$

and

$$\Phi^* = \Phi_0 + \sigma_u^{-2}U'U$$

with ϕ_0 being the mean vector and Φ_0 being the precision matrix for the prior distribution of ϕ.[25]

A reason to use an autoregressive model is that many times households will purchase a large quantity of beef products for one four-week period and then prepare it over more than one time period. This would cause the errors to be correlated across time.

Non-normal Error and Variable Transformation

Yen and Jensen (1995) make two suggestions to help correct for the heteroscedasticity problems in Tobit models. They state that heteroscedastic errors are usually prevalent in data food demand analysis. They suggest non-normal error structures and variable transformations. With non-normal error structure, the priors would also have to have different distributions in order to be conjugate priors. As long as they are conjugate, the conditional distributions will be easy to derive. The problem arises in that there are not many conjugate families available.

With variable transformation, a variance-stabilizing transformation is used to correct the heteroscedasticity problem (Abraham and Ledolter 1983). The process for doing this is as follows. Let $\eta_{it} = \mathrm{E}(Y_{it})$ and assume that the variance of ε_{it} is functionally related to η_{it} according to

$$\mathrm{Var}(Y_{it}) = [h(\eta_{it})]^2\, \sigma^2$$

where h is some known function. The objective is to find a transformation of Y_{it}, $g(Y_{it})$ that stabilizes the variance. Expanding $g(Y_{it})$ in a first-order Taylor series around η_{it} results in

$$g(Y_{it}) \cong g(\eta_{it}) + (Y_{it} - \eta_{it})\, g'(\eta_{it}).$$

The resulting variance of $g(Y_{it})$ is

$$\mathrm{Var}[g(Y_{it})] \cong [g'(\eta_{it})]^2\, [h(\eta_{it})]^2\, \sigma^2.$$

So to correct for the heteroscedasticity, g must be chosen so that

$$g'(\eta_{it}) = 1/\, h(\eta_{it}).$$

Testing Structural Change over Time

Another extension to the model could be testing structural change over time, i.e. the parameters' true values change over time. One such model is the Cooley-Prescott model (Kinnucan and Venkateswaran 1994). In this model, the fixed effects parameters (β_t) change over time. The model is as follows:

$$Y_{it}^{*} = \alpha_i + \beta_t' x_{it};\ \alpha_i \sim N(0, \sigma_\alpha^2)$$

where $\beta_t = \beta_t^P + u_t;\ u_t \sim N(0, \sigma_u^2)$ and $\beta_t^P = \beta_{t-1}^P + v_t;\ v_t \sim N(0, \sigma_v^2)$. The β_t are now time-varying random variables, and they are based on past values of β.

Kinnucan and Venkateswaran believe that using a time-varying parameter allows for greater realism in capturing the market response to a generic advertising campaign such as that used in the Grand Junction experiment. They state that econometric models that do not use these time-varying parameters are inappropriate for long-term policy evaluation. New policies and decisions can cause changes in these parameters

Revision of "Featured" Items Variable

In the model, the FEAT variable is used in an attempt to control for the period to period variation in the intensity of non-television advertising. After further review, it appears that this variable does not fully capture the intended effect. As defined, FEAT could vary due to variation across households in preferences toward purchase of featured items even if the local print and in-store display advertising intensities remained unchanged. An alternative to this variable is the variable used by Jensen and Schroeter that measures the proportion of each period's total panel expenditures on beef that were made on "featured" items (their "PRPFT" variable). This alternative might have better served as a rough measure of the intensity of non-television advertising for the test area. Re-estimation of the model with "PRPFT" replacing "FEAT" was not undertaken for this report, however, due to the significant computer time costs that would have been involved.

Jensen and Schroeter found that an increase in the PRPFT variable has a significant positive effect on demand for fresh beef in the aggregate. It is expected that this would also be true for the demand for each type of fresh beef.

Re-estimating with the new variable will cause a change in the posterior means of the other parameters. While this is a major concern, because of the high number of regressors in the model, the chances of this one variable having a major impact on the other variables is small.

Approaches to Incorporating Advertising

Other possible revisions involve modifications of the approach used in incorporating advertising. The two that will be discussed are panel-phase interaction dummy variables and a 12-month, fourth-order distributed lag of exposure levels.

The panel-phase interaction approach is a simpler way of incorporating advertising's effect. In addition to the phase and panel dummy variables that are already included in the model, dummy variables would be added for the panel-phase interactions. The variables are defined as follows:

BAP_PHS1_{it} = a dummy variable equal to 1 if household i is in the base ad panel and if period t is in phase 1 of the advertising test,

BAP_PHS2_{it} = a dummy variable equal to 1 if household i is in the base ad panel and if period t is in phase 2 of the advertising test,

HAP_PHS1_{it} = a dummy variable equal to 1 if household i is in the heavy ad panel and if period t is in phase 1 of the advertising test,

HAP_PHS2_{it} = a dummy variable equal to 1 if household i is in the heavy ad panel and if period t is in phase 2 of the advertising test.

As before, the BAP variable would pick up the time invariant effect of base-ad panel membership and the PHS1 variable would reflect any phase 1 effect that is common across panels. The BAP_PSH1 variable would pick up whatever effect is unique to those in the base-ad panel during phase 1--presumably the impact of the test advertising telecast to base-ad panel households during this period. The main drawback of this approach is that it does not take into account the variation in advertising intensity within a given phase of the experiment.

The second approach is one that was also used by Jensen and Schroeter. It is a 12-month, fourth-order distributed lag in advertising intensities in which the lag weights are estimated by the Almon polynomial technique:

$$\sum_{j=0}^{11} w_j GRP_{i,t-j} = \sum_{j=0}^{11} (\alpha_0 + \alpha_1 j + \alpha_2 j^2 + \alpha_3 j^3 + \alpha_4 j^4) GRP_{i,t-j} .$$

Jensen and Schroeter found that, with regard to advertising's effects, the implications of the fourth-order Almon polynomial specification were very similar to those of the second-order fixed weight (Ward and Dixon) specification.

CONCLUSION

This research was conducted to test the effect of television advertising on the demand for different types of fresh beef products. The data are from a marketing research experiment done in Grand Junction, Colorado from 1985 to 1987. The experiment utilized cable television test advertisements and supermarket scanner data on panel households' beef purchases.

The model used to analyze the data is a random effects Tobit model. This is used because many observations of the dependent variables are at zero values, so a standard GLS model is not appropriate. Likewise, a conventional Tobit model fails to allow for the household specific effects one might expect to find in a panel data study. Because of the computational difficulties in finding maximum likelihood estimates of the random effects Tobit model, a Bayesian posterior simulation technique utilizing Gibbs sampling is used. The technique uses sequential sampling from conditional distributions of the parameters to simulate the joint posterior distribution of the model's parameters.

In the model, many other variables in addition to advertising are included. Prices of beef and other fresh meats, demographic variables, and an income variable are among those included to control for other sources of variation in household beef demand.

Drawbacks of the data set and possible extensions to the model are also presented. The data set also has a big advantage. Most advertising studies are based on aggregate data. Because of the household specific nature of this data, a more extensive analysis of the demographic effects is possible.

The effect of advertising, represented by a 12-month, second-order distributed lag in advertising intensities, is positive for all three categories, but the posterior probability for the positive advertising effect on roast beef demand is not as high as that on steak and ground beef.

APPENDIX

FORTRAN PROGRAM FOR GIBBS SAMPLER

```
      program main
      parameter (kk=28, nn=33350, tt=23, ii=1450)
      real*8 primean(kk), a(kk,kk), valp, vu, s2alp, s2u, alpha(nn),
     *   beta(kk), sigu, sigalp, ystar(nn), y(nn), x(nn,kk)
      integer irem, nseed, nburn, ngibbs, n, k, i
      character*80 filepar
      external g05cbf
c
      write(*,*) 'Enter name of dependent variable file'
      read(*,800) filein1
      open(unit=11, file=filein1, status='old')
c
      do 100 n=1,nn
        read(11,801) y(n)
100   continue
c
      write(*,*) 'Enter name of independent variables file'
      read(*,800) filein2
      open(unit=12, file=filein2, status='old')
c
      do 200 n=1,nn
        read (12,802) (x(n,k), k=1,kk)
200   continue
c
      write(*,*) 'Enter name of output files for parameter draws'
      read(*,800) filepar
      open(unit=21, file=filepar, status='unknown')
      write(*,*) 'Enter value of random number seed'
      read(*,803) nseed
      write(*,*) 'Input burn-in'
      read(*,*) nburn
      write(*,*) 'Input number of Gibbs loops'
      read(*,*) ngibbs
c
      call g05cbf(nseed)
      call prior(primean,a,valp,vu,s2alp,s2u)
      call initial(alpha,beta,sigu,sigalp,primean,s2u,s2alp)
c
      do 1 i=1,ngibbs
        irem=mod(i,10)
        call star(ystar,alpha,beta,sigu,y,x)
        call dalpha(ystar,alpha,beta,sigu,sigalp,x)
        call drawbeta(ystar,alpha,beta,sigu,x,primean,a)
        call dsigma(ystar,alpha,beta,sigu,sigalp,x,valp,vu,s2alp,s2u)
c
        if(irem .eq. 0) write (*,*) i
        if (i .gt. nburn) then
          if (irem .eq.0) then
          write (21,805) i,(beta(k),k=1,kk), (alpha(n),n=1,nn,tt),
     *      sigalp, sigu
          endif
        endif
```

```
1     continue
c
800   format(a80)
801   format(f6.3)
802   format(5(f6.2,1x),4(f2.0,1x),3(f1.0,1x),f6.3,2(f1.0,1x),2(f2.0,1x),
     *  8(f1.0,1x),2(f8.2,1x),f7.3)
803   format(i5)
805   format(i5,1x,28(f14.9,1x),1450(f14.9,1x),2(f14.9,1x))
c
      stop
      end
c-----------------------------------------------------------------------
c This subroutine sets the value of the prior parameters.
c-----------------------------------------------------------------------
c
      subroutine prior(primean,a,valp,vu,s2alp,s2u)
      parameter (kk=28)
      real*8 primean(kk), a(kk,kk), valp, vu, s2alp, s2u
      integer k,j
c
      do 1 k=1,kk
        primean(k)=0
1     continue
c
      do 2 k=1,kk
        do 3 j=1,kk
          if (k .eq. j) then
            a(k,j)=0.0001d0
          else
            a(k,j)=0.0d0
          endif
3       continue
2     continue
c
      valp=1.0d0
      vu=1.0d0
      s2alp=0.5d0
      s2u=0.5d0
c
      return
      end
c----------------------------------------------------------------------------
c This subroutine intializes the Markov chain for each parameter, including
c the states.  Initial values are equal to prior means.
c----------------------------------------------------------------------------
c
      subroutine initial(alpha,beta,sigu,sigalp,primean,s2u,s2alp)
      parameter (kk=28, nn=33350)
      real*8 alpha(nn), beta(kk), sigu, sigalp, primean(kk), s2u, s2alp
      integer k,n
c
      do 1 n=1,nn
```

```
        alpha(n)=0.0d0
1     continue
c
      do 2 k=1,kk
        beta(k)=primean(k)
2     continue
c
      sigu=dsqrt(s2u)
      sigalp=dsqrt(s2alp)
c
      return
      end
c---------------------------------------------------------------------
c This subroutine draws latent dependent variables for the observations in
c which the dependent variable is equal to zero.  The distribution
c conditioned on everything else is a truncated normal distribution.  This
c value is assigned to ystar.  If the dependent variable is not equal to
c zero then that value is assigned to ystar.  The distribution of ystar then
c becomes normal.
c----------------------------------------------------------------------
c
      subroutine star(ystar,alpha,beta,sigu,y,x)
      parameter (nn=33350,kk=28)
      real*8 ystar(nn), alpha(nn), beta(kk), sigu, x(nn,kk), y(nn), aa,
     *  b, mu, xbeta, unif, c, d, xvec(kk), g05daf, f06eaf, s15abf,
     *  g01faf
      integer n, k, incx, incy, ifail1, ifail2, unilow
      character*1 tail
      external g05daf, f06eaf, s15abf, g01faf
c
      do 1 n=1,nn
        do 2 k=1,kk
          xvec(k)=x(n,k)
2       continue
        if (y(n) .eq. 0) then
          incx=1
          incy=1
          xbeta=f06eaf(kk,xvec,incx,beta,incy)
          mu=alpha(n)+xbeta
          d=mu/sigu
c
          ifail1=0
          aa=s15abf(d,ifail1)
          if (ifail1 .ne. 0) then
            write(*,*) 'error occured in function s15abf'
          endif
c
          b=1.0d0-aa
          unilow=0.0d0
          unif=g05daf(unilow,b)
          tail='l'
c
```

```
          ifail2=0
          if (unif .le. 0.0d0) then
            unif=0.1d-15
          endif
          if(unif .ge. 1.0d0) then
            unif=1 - 0.1d-15
          endif
          c=g01faf(tail,unif,ifail2)
          if (ifail2 .ne. 0) then
            write(*,*) 'error occured in function g01faf'
          endif
c
          ystar(n)=c*sigu+mu
        else
          ystar(n)=y(n)
        endif
1     continue
c
      return
      end
c-----------------------------------------------------------------------
c This subroutine draws values of the alpha vector from the conditional
c distribution given everything else.  This conditional distribution is
c multivariate normal.  I assigned a temporary vector with one element
c for each household.  Then each value was assigned to each time period
c resulting in an aplha vector with one element for each observation.
c-----------------------------------------------------------------------
c
      subroutine dalpha(ystar,alpha,beta,sigu,sigalp,x)
      parameter (tt=23,ii=1450,nn=33350,kk=28)
      real*8 ystar(nn), alpha(nn), beta(kk), sigu, sigalp, x(nn,kk),
     *  sumy, sumx(kk), xbeta, mu, var, sdev, tempal(ii), f06eaf,
     *  g05ddf
      integer n, k, i, t, j, m, g, h, incx, incy
      external f06eaf, g05ddf
c
      g=1
      h=tt
      do 1 i=1,ii
        sumy=0.0d0
        do 2 n=g,h
          sumy=sumy+ystar(n)
2       continue
c
        do 3 k=1,kk
          sumx(k)=0.0d0
          do 4 n=g,h
            sumx(k)=x(n,k)+sumx(k)
4         continue
3       continue
c
        incx=1
```

```
        incy=1
        xbeta=f06eaf(kk,sumx,incx,beta,incy)
        mu=(sumy-xbeta)/(tt+sigu**2/sigalp**2)
        var=1.0d0/((tt/sigu**2)+(1.0d0/sigalp**2))
        sdev=dsqrt(var)
        tempal(i)=g05ddf(mu,sdev)
        g=h+1
        h=h+tt
1     continue
c
      j=1
      m=tt
      do 5 i=1,ii
        do 6 n=j,m
          alpha(n)=tempal(i)
6       continue
        j=m+1
        m=m+tt
5     continue
c
      return
      end
c-----------------------------------------------------------------------
c This subroutine draws the values of the beta vector from the conditional
c distribution given everything else.  This conditional distribution is
c mulitvariate normal.  The mean is the weighted average of the least
c squares estimator, regressing (y - alpha) on x, and the prior mean.
c The subroutines called in this subroutine are from the nag subroutine
c library.
c-----------------------------------------------------------------------
c
      subroutine drawbeta(ystar,alpha,beta,sigu,x,primean,a)
      parameter (nn=33350,kk=28,lmove=16689,nr=435)
      real*8 ystar(nn), alpha(nn), beta(kk), sigu, x(nn,kk),
     *  primean(kk), a(kk,kk), xp(nn,kk), xpx(kk,kk), z(nn),
     *  xpxa(kk,kk), mu(kk), var(kk,kk), yalp(nn), work(kk),
     *  xpxainv(kk,kk), xpyalp(kk), abetabar(kk), xpyaabb(kk),
     *  r(nr), al, be, eps
      integer n, k, nnkk, move(lmove), ifail1, ifail2, ifail3,
     *  ifail4, opt, j, ipiv(kk), info1, info2, incx,
     *  incy
      character*1 trans
      external f01crf, f01ckf, f07ajf, f06paf, g05ezf, g05eaf, f07adf
c
      do 1 k=1,kk
        do 2 n=1,nn
          xp(n,k)=x(n,k)
2       continue
1     continue
c
      ifail1=0
      nnkk=nn*kk
```

```
      call f01crf(xp,nn,kk,nnkk,move,lmove,ifail1)
      if (ifail1 .ne. 0) then
        write(*,*) 'error occured in subroutine f01crf'
      endif
c
      opt=1
      ifail2=0
      call f01ckf(xpx,xp,x,kk,kk,nn,z,nn,opt,ifail2)
      if (ifail2 .ne. 0) then
        write(*,*) 'error occured in subroutine f01ckf'
      endif
c
      do 3 k=1,kk
        do 4 j=1,kk
          xpxa(j,k) = xpx(j,k)/sigu**2+a(j,k)
4       continue
3     continue
c
      do 5 j=1,kk
        do 6 k=1,kk
          xpxainv(j,k)=xpxa(j,k)
6       continue
5     continue
c
      call f07adf(kk,kk,xpxainv,kk,ipiv,info1)
      if (info1 .ne. 0) then
        write(*,*) 'error occured in subroutine f07adf'
      endif
c
      call f07ajf(kk,xpxainv,kk,ipiv,work,kk,info2)
      if (info2 .ne. 0) then
        write(*,*) 'error occured in subroutine f07ajf'
      endif
c
      do 7 n=1,nn
        yalp(n)=ystar(n) - alpha(n)
7     continue
c
      al=1.0d0
      be=0.0d0
      incx=1
      incy=1
c
      trans='t'
      call f06paf(trans,nn,kk,al,x,nn,yalp,incx,be,xpyalp,incy)
c
      trans='n'
      call f06paf(trans,kk,kk,al,a,kk,primean,incx,be,abetabar,incy)
c
      do 8 k=1,kk
        xpyaabb(k)=xpyalp(k)/sigu**2+abetabar(k)
8     continue
```

```
c
      trans='n'
      call f06paf(trans,kk,kk,al,xpxainv,kk,xpyaabb,incx,be,mu,incy)
c
      do 9 j=1,kk
        do 10 k=1,kk
          var(j,k)=xpxainv(j,k)
10      continue
9     continue
c
      eps=0.0d0
      ifail3=0
      ifail4=0
c
      call g05eaf(mu,kk,var,kk,eps,r,nr,ifail3)
      if (ifail3 .ne. 0) then
        write(*,*) 'error occured in subroutine g05eaf'
      endif
c
      call g05ezf(beta,kk,r,nr,ifail4)
      if (ifail4 .ne. 0) then
        write(*,*) 'error occured in subroutine g05ezf'
      endif
c
      return
      end
c-----------------------------------------------------------------------------
c This subroutine draws values for the variances of the random effects and
c error terms from the conditional distribution given everything else.
c The conditional distributions are inverse gamma.  This is first done by
c drawing values from the gamma distribution and taking the inverse of those
c values.
c-----------------------------------------------------------------------------
c
      subroutine dsigma(ystar,alpha,beta,sigu,sigalp,x,valp,vu,s2alp,
     *  s2u)
      parameter (ii=1450,tt=23,kk=28,nn=33350)
      real*8 ystar(nn), alpha(nn), beta(kk), sigu, sigalp, x(nn,kk),
     *  valp, vu, s2alp, s2u, sum, u2, sumal2, aalpha, balpha, au,
     *  bu, sig2ai(1), sig2ui(1), sig2al, sig2u, f06eaf, xvec(kk)
      integer n, i, t, k, ifail1, ifail2, incx, incy, p
      external f06eaf, g05fff
c
      sum=0.0d0
      do 1 n=1,nn
        do 2 k=1,kk
          xvec(k)=x(n,k)
2       continue
c
        incx=1
        incy=1
        xbeta=f06eaf(kk,xvec,incx,beta,incy)
```

```
          u2=(ystar(n)-alpha(n)-xbeta)**2
          sum=sum+u2
 1      continue
 c
        sumal2=0.0d0
        do 3 n=1,nn,tt
          sumal2=sumal2+(alpha(n))**2
 3      continue
 c
        aalpha=(valp+ii-1.0d0)/2.0d0
        balpha=2.0d0/(sumal2+valp*s2alp)
        if (balpha .le. 0.0d0) then
          balpha=0.1d-15
        endif
        au=(ii*tt+vu-1.0d0)/2.0d0
        bu=2.0d0/(sum+vu*s2u)
        if (bu .le. 0.0d0) then
          bu=0.1d-15
        endif
 c
        p=1
        ifail1=0
        call g05fff(aalpha,balpha,p,sig2ai,ifail1)
        if (ifail1 .ne. 0) then
          write(*,*) 'error occured in subroutine g05fff'
        endif
 c
        ifail2=0
        call g05fff(au,bu,p,sig2ui,ifail2)
        if (ifail2 .ne. 0) then
          write(*,*) 'error occured in subroutine g05fff'
        endif
 c
        sig2al=1.0d0/sig2ai(1)
        sig2u=1.0d0/sig2ui(1)
        sigalp=dsqrt(sig2al)
        sigu=dsqrt(sig2u)
 c
        return
        end
```

NOTES

1. Jensen and Schroeter reported the results of random effects and Tobit models separately but did not attempt to treat both issues in a single analysis.

2. The reason for using only these types of beef is that they account for over 95% of consumer expenditures on beef (Heien and Pompelli 1988).

3. "Gross rating points are computed as the sum of all commercial break ratings for breaks in which the advertisements appeared. Break ratings are the averages of the quarter-hour program ratings (the percentage of television households viewing the program) for programs on either side of the break" (Jensen and Schroeter 1992).

4. The data actually were sufficiently detailed to permit a greater disaggregation by product type. Analysis was limited to these three categories because of their importance (see note 2) and to keep the problem manageable.

5. Some problems with and alternatives to the Tobit model will be discussed briefly in the "Possible Extensions" section.

6. A random effects model is chosen over a fixed effects model because the idiosyncratic behavior of individual households is not of interest in this study.

7. The dependent variables in this model are the quantities of particular types of fresh beef purchased by household i in period t (adjusted as in the definition given in the model variables section).

8. The explanatory variables represent prices, household demographics, income levels, and advertising intensities.

9. Other posterior simulators include the acceptance method, independence sampling, the Metropolis algorithm, and the Metropolis-Hastings algorithm (Geweke 1995).

10. The inverse CDF method begins with a pseudo-random sequence $\{u_i\}$ in which $u_i \sim$ i.i.d. Unif(0,1). Once $\{u_i\}$ is generated, then the realizations u can be used to generate random numbers from any one-to-one univariate distribution. Suppose that x is continuous, and the inverse CDF of X: $F^{-1}(p) = \{c\colon P(X \le c) = p\}$ exists. Then x and $F^{-1}(u)$ have the same distribution.

11. The Numerical Algorithms Group (NAG) library contains FORTRAN sub-routines to perform numerical and statistical analysis. Examples of routines used in this program include matrix manipulation and simulating random values from statistical distributions.

12. The value of k is chosen to obtain essentially uncorrelated draws. The procedure for this is explained in the Convergence Results chapter.

13. Due to the large size of the data set, results should be relatively invariant with respect to prior means on β.

14. The variance-covariance matrix of β is Λ^{-1}, so Λ is the precision matrix. The diagonal elements of Λ are set very low, and the off-diagonal elements are zero.

15. Defined in this way, the total number of standard people in all of the households is equal to the total headcount.

16. It is an unobserved opportunity wage rate for unemployed individuals which measures the wage rate for which that individual would supply his or her labor.

17. $w_i^* = w_{11-i}^*$, w_0^*=0.033, w_1^*=0.0604, w_2^*=0.0824, w_3^*=0.0989, w_4^*=0.1099, and w_5^*=0.1154.

18. The parameter sampled is actually the square root of the variance.

19. The fact that the draws are correlated is not relevant to any inferences made in the empirical results chapter.

20. Credible intervals are computed by taking percentiles of the posterior draws of the Gibbs sample. All of the credible intervals used in this analysis are 90% credible intervals; therefore, the 5th percentile is the value for the lower limit of the interval and the 95th percentile is the value for the upper limit of the interval.

21. A "partial" elasticity is an elasticity that is evaluated at constant food expenditures compared to total elasticity that is evaluated at constant income levels. For a complete discussion, refer to Jensen and Schroeter.

22. The elasticities in each of these studies are conditional on positive purchase quantities. To compute unconditional elasticities, these elasticities need to multiplied by the probability of positive purchase.

23. By significance, it is meant that the posterior probability of being positive is greater than .90 or less than .10.

24. The precision matrix is the inverse of the variance-covariance matrix.

25. The prior distribution of ϕ is given by $p(\phi) \sim \mathrm{MVN}(\phi_0, \Phi_0^{-1})$.

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